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Cubic Averaging Aggregation Operators with Multiple Attributes Group Decision Making Problem

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Abstract

The group decision making is a very useful technique for ranking the group of alternatives, the cubic averaging operator is new tool in group decision making problems. In this article, we develop a series of new operators so called cubic weighted averaging *(CWA)* operator, cubic ordered weighted averaging *(COWA)* operator and cubic hybrid averaging *(CHA)* operator. We also discussed some particular cases and properties of these operators. Furthermore, we apply the proposed aggregation operators to deal with multiple attribute group decision making in which decision information takes the form of cubic numbers. Finally, we used some practical examples to illustrate the validity and feasibility of the proposed methods by comparing with other methods.

Keywords: Cubic numbers; Aggregation operators; Group decision making

Introduction

The idea of fuzzy set theory was developed by LA Zadeh [1]. In fuzzy set theory the degree of member ship function was discussed. Fuzzy set theory has been studied in multi directions such that, medical diagnosis, computer computer science, artificial intelligence, operation research, management science, control engineering, robotics, expert systems and decision making problems. Later on the great idea of intuitionistic fuzzy set (*IFS*) theory was developed by Atanassov [2], and discussed the degree of membership and non-membership function. *IFS* is the generalization of fuzzy set theory. There are many advantage of *IFS* theory such as, using engineering, management science, computer science. Fuzzy set discuss the case in which the membership involve, intuitionistic fuzzy sets *(IFS)* is generalization of fuzzy set, the uncertainty problem does not explain by means of intuitionistic fuzzy set.

So therefore Jun *et al.* [3] define the concept of cubic set, Jun define a new theory which is known as cubic set theory. This theory is able to deal with uncertain problem. Cubic set theory also explains the, satisfied, unsatisfied, and uncertain information. While fuzzy set theory and intuitionistic fuzzy set fail to explain these terms. The cubic set is a generalization of fuzzy set and intuitionistic fuzzy set. Cubic set is collection of interval value fuzzy set *(IVFS)* and fuzzy set, while *(IFS)* is only fuzzy set. Cubic set more desirable the information then fuzzy set and intuitionistic fuzzy set. After that, Mahmood *et al.* [4] defined cubic hesitant fuzzy sets *(CFHSs)* by combining interval valued hesitant fuzzy sets *(IVHFSs) (2013)* and hesitant fuzzy sets *(HFSs) (2009)* and defined some basic operations, properties, of cubic hesitant fuzzy sets.

The concept of neutrosophic set *(NS)* developed by Smarandache [5] and Chang Su Kim and Smarandache [6] is a more general platform which extends the concepts of the classic set and fuzzy set, intuitionistic fuzzy set and interval valued intuitionistic fuzzy set. Neutrosophic set theory is applied to various part which extend the concept of cubic sets to the neutrosophic sets. They introduced the notions of truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic sets and truth-external (indeterminacy-external, falsity-external) neutrosophic cubic sets, and investigate related properties. Such that the P-union and the P-intersection of truth-internal (indeterminacy-internal, falsity- internal) neutrosophic cubic sets are also truth-internal (indeterminacy-internal, falsity-internal) neutrosophic cubic sets. Jun *et al.* [7] also worked on neutrosophic cubic sets and developed its various properties.

Information aggregation is an important research topic in many applications such as fuzzy logic systems and multi-attribute decision making as discussed by Chen and Hwang [8]. Research on aggregation operators has received increasing attention as shown in the literature. Research papers of Yager and Kacprzyk, Yager [9-11], Calvo *et al.* [12], Xu and Da [13], Chen and Chen [14], and others are included in this area. Research on aggregation methods with intuitionistic fuzzy or intervalvalued intuitionistic fuzzy information is also active. Mitchell [15] defined an intuitionistic ordered weighted averaging *(OWA)* operator to integrate several *IFS* and presented a simple application of the new intuitionistic *OWA* operator in multiple-expert multi-criteria decision making. Xu [16] proposed intuitionistic fuzzy ordered weighted averaging *(IFOWA)* operator and intuitionistic fuzzy hybrid averaging *(IFHA)* operator. Xu and Yager [17] developed several geometric aggregation operators, such as intuitionistic fuzzy weighted geometric *(IFWG)* operator, intuitionistic fuzzy ordered weighted geometric *(IFOWG)* operator, and intuitionistic fuzzy hybrid geometric *(IFHG)* operator, and presented an application of the *IFHG* operator to multi-attribute group decision making *(MAGDM)* with intuitionistic fuzzy information

Due to the motivation and inspiration of the above discussion, we generalized the concept of Pythagorean trapezoidal linguistic fuzzy set which give us more accurate and precious result as compare to the above mention operators. Thus keeping an advantage of the above mention aggregation operators. In this paper, we develop a series of cubic aggregation operators such that cubic weighted averaging *(CWA)* operator, cubic ordered weighted averaging *(COWA)* operator and cubic hybrid averaging *(CHA)* operator.

The rest of this paper is structured as follows. In section 2, we review some basic Definition and operational laws which are used our latter work. In section 3, we analyze different types of cubic weighted averaging *(CWA)* operator, cubic ordered weighted averaging *(COWA)* operator and cubic hybrid averaging *(CHA)* operator and also study its various properties [18-21]. In section 4 briefly describe the decision making process based on developed operators and we give a numerical example in section 5. In section 6 summarizes the main conclusions of the paper.

Preliminaries

Definition 1. [2] *Let a set X be fixed. An (IFS) A in X is an object having the form*:

$$A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle \mid x\in X\right\} \qquad (1)$$

where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to *A* respectively, and for every

$$x\in X \ : 0\leq \mu_A(x)+\nu_A(x)\leq 1 \qquad (2)$$

For each *(IFS) A* in *X*, if

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x),\ \text{for all } x\in X \qquad (3)$$

then $\pi_A(x)$ is called the degree of indeterminacy of *x* to *A*.

Definition 2. [9] *Let X be a fixed non empty set. A cubic set is an object of the form,*

$$C=\{\langle a, A(a), \lambda(a)\rangle \ : \ a\in X\},$$

where A is an (IVFS) and λ *is a fuzzy set in X. A cubic set* $C \quad \langle a, A(a), \quad (a)\rangle$ *is simply denoted by* $\tilde{C}=\langle \tilde{A},\lambda\rangle=\left\langle [\bar{a},a^{+}],\lambda\right\rangle$. *The collection of all cubic set is denoted by C(X).*

(a) if $\lambda\in\tilde{A}(x)$ for all $x\in X$ so it is called interval cubic set;

(b) If $\lambda\notin\tilde{A}(x)$ for all $x\in X$ so it is called external cubic set;

(c) If $\lambda\in\tilde{A}(x)$ or $\lambda\notin\tilde{A}(x)$ its called cubic set for all $x\in X$.

Definition 3. [9] *Let* $A=\langle\tilde{A},\lambda\rangle$ *and* $B=\langle\tilde{B},\mu\rangle$ *be cubic set in* X *, then we define*

(a) (Equality) $A=B \Leftrightarrow A=B$ and $\lambda=\mu$.

(b) ($P-$order) $A\subseteq_A B \Leftrightarrow A\subseteq B$ and $\lambda\leq\mu$.

(c) ($R-$order) $A\subseteq_R B \Leftrightarrow A\subseteq B$ and $\lambda\leq\mu$.

Definition 4. [9] *The complement of* $A=\langle\tilde{A},\lambda\rangle$ *is defined to be the cubic set*

$$A^c=\{\langle x, A^c(x), 1-\lambda(x)\rangle \mid x\in X\}.$$

Cubic Numbers, Score and Accuracy Function

In this section we define the some operational laws of cubic numbers. We define score function and accuracy function of a cubic set.

Definition 5. *Let* $C = \langle \tilde{A}_c, \lambda_c \rangle$, $C_1 = \langle \tilde{A}_{c_1}, \lambda_{c_1} \rangle$, *and* $C_2 = \langle \tilde{A}_{c_2}, \lambda_{c_2} \rangle$, *be three (CVs) , then the following operational laws holds.*

1. $C_1 \oplus C_2 = \left\langle \left[\bar{a}_{c_1} + \bar{a}_{c_2} - \bar{a}_{c_1} \bar{a}_{c_2},\ a^+_{c_1} + a^+_{c_2} - a^+_{c_1} a^+_{c_2} \right], \lambda_{c_1}\lambda_{c_2} \right\rangle$;

2. $C_1 \otimes C_2 = \left\langle \left[\bar{a}_{c_1} \bar{a}_{c_2},\ a^+_{c_1} a^+_{c_2} \right], \lambda_{c_1} + \lambda_{c_2} - \lambda_{c_1}\lambda_{c_2} \right\rangle$;

3. $\delta C = \left\langle \left[1-(1-\bar{a}_c)^\delta\ ,(1-(1- a^+_c)^\delta \right], \lambda_c^\delta \right\rangle, \qquad \delta \geq 0$;

4. $C^\delta = \left\langle \left[(\bar{a}_c)^\delta\ ,\ (a^+_c)^\delta \right], 1-(1-\lambda_c)^\delta \right\rangle, \qquad \delta \geq 0.$

Based on the *(CVs)* We introduce a score function *S* to evaluate the degree of suitability that an alternative satisfies a decision maker's requirement. Let $C = \langle \tilde{A}_c, \lambda_c \rangle$ be an CV, where

$$\tilde{A}_c \in [0,1],\ \lambda_c \in [0,1]$$

The score of C can be evaluated by the score function S shown as

$$s(C) = \frac{\tilde{A}_c - \lambda_c}{3} = \frac{\bar{a} + a^+ - \lambda}{3} \qquad (4)$$

Now an accuracy function to evaluate the degree of accuracy of the *(CV)* $C = \langle \tilde{A}_c, \lambda_c \rangle$ as

$$h(C) = \frac{1 + \tilde{A}_c - \lambda_c}{3} = \frac{1 + \bar{a} + a^+ - \lambda}{3} \qquad (5)$$

where $h(C) \in [0,1]$.

The larger the value of *h(C)* , the higher the degree of accuracy of the degree of membership of the *(CV)* . Then, utilized the score function, the accuracy function, and the minimum and maximum operations to develop another technique for handling multiple attribute decision-making problems based on cubic information.

Remarks

1. If *s(C) < s(D)*,then *C* is smaller than *D* , denoted by *C < D*;
2. If *s(C) = s(D)*,then we have;
 i. If *h(C) = h(D)*,then *C* and *D* represent the same information, denoted by *C = D*;
 ii. If *h(C) < h(D)*,then *C* is smaller than *D*, denoted by *C < D*.

(CWA,COWA and CHA) Operators

Definition 6. *Let* Ω *be the set of all cubic values and* $C_j = \langle \tilde{A}_{c_j}, \lambda_{c_j} \rangle = \langle [\bar{a}_{c_j}, a^+_{c_j}], \lambda_{c_j} \rangle$

(*j*=1,2,...,*n*) be a collection of cubic values, and let $CWA : \Omega^n \to \Omega$, if

$$CWA(c_1, c_2, ..., c_n) = w_1 c_1 \oplus w_2 c_2 \oplus ... \oplus w_n c_n$$
$$= \left(\begin{array}{c} [1 - \prod_{j=1}^{n} (1 - \bar{a}_{c_j})^{w_j},\ 1 - \prod_{j=1}^{n} \left(1 - a^+_{c_j})^{w_j}\right)], \\ \prod_{j=1}^{n} \left(\lambda_{c_j}\right)^{w_j} \end{array} \right) \qquad (6)$$

where $w=(w_1,w_2,...,w_n)^T$ is weight vector of C_j *(j=1,2,...,n) with* $w_j \in [0,1]$ *and* $\sum_{j=1}^{n} w_j = 1.$

Example 1. *Let* $C_1 = \langle [0.3, 0.6], 0.6 \rangle, C_2 = \langle [0.6, 0.7], 0.4 \rangle, C_3 = \langle [0.1, 0.5], 0.3 \rangle$ *and* $C_4 = \langle [0.2, 0.3], 0.5 \rangle$ *be four cubic values, and suppose that* $w = (0.30, 0.20, 0.40, 0.10)^T$ *be the weight vector of* C_j *(j=1,2,...,4) . Using equation* (6) *such that*

$$CWA(c_1,c_2,...,c_4)=\left(\begin{array}{c}[1-\prod_{j=1}^{4}(1-a^{-}_{c_j})^{w_j},\ 1-\prod_{j=1}^{4}\left(1-\ a^{+}_{c_j})^{w_j}\right)],\\ \prod_{j=1}^{4}\left(\lambda_{c_j}\right)^{w_j}\end{array}\right),$$
$$=\langle[0.29,0.56],0.41\rangle.$$

Definition 7. *Let COWA operator of dimension n* is a mapping $COWA\ :\ \Omega^n\rightarrow\Omega,$ that has an associated vector $w=(w_1,w_2,...,w_n)^T$ *is weight vector of* C_j *(j=1,2,..., n) with* $w_j\in[0,1]$ *and* $\sum_{j=1}^{n}w_j=1,$ *such that*

$$COWA(c_1,c_2,...,c_n)=w_1c_{\sigma_{(1)}}\oplus w_2c_{\sigma_{(2)}}\oplus...\oplus w_nc_{\sigma_{(n)}}$$
$$=\left(\begin{array}{c}[1-\prod_{j=1}^{n}(1-a^{-}_{c_{\sigma(j)}})^{w_j},\ 1-\prod_{j=1}^{n}\left(1-\ a^{+}_{c_{\sigma(j)}})^{w_j}\right)],\\ \prod_{j=1}^{n}\left(\lambda_{c_{\sigma(j)}}\right)^{w_j}\end{array}\right)\qquad(7)$$

where $c_{\sigma(j)}$ *is the jth largest of the* c_j. *Especially, if* $w=\left(\frac{1}{n},\frac{1}{n},...,\frac{1}{n}\right)^T$ *, then the COWA operator is reduced to the CA operator.*

Example 2. Let $C_1=\langle[0.3,0.5],0.2\rangle,C_2=\langle[0.2,0.4],0.1\rangle,C_3=\langle[0.6,0.7],0.5\rangle,\ C_4=\langle[0.4,0.8],0.5\rangle$ *and* $C_5=\langle[0.1,0.3],0.3\rangle$ *be five cubic values, and suppose that* $w=(0.25,0.15,0.10,0.20,0.30)^T$ *be the weight vector of* C_j *(j=1,2,...,5) . First of all we find the score function by applying E.q* (4) *such that*

$$S(c_1)=\frac{0.3+0.5-0.2}{3}=0.20,\ S(c_2)=\frac{0.2+0.4-0.1}{3}=0.16$$
$$S(c_3)=\frac{0.6+0.7-0.5}{3}=0.26,\ S(c_4)=\frac{0.4+0.8-0.5}{3}=0.23$$
$$S(c_5)=\frac{0.1+0.3-0.3}{3}=0.03.$$

Since $S(c_3)>S(c_4)>S(c_1)>S(c_2)>S(c_5)$ Hence

$$C_{\sigma(1)}=\langle[0.6,0.7],0.5\rangle,C_{\sigma(2)}=\langle[0.4,0.8],0.5\rangle,$$
$$C_{\sigma(3)}=\langle[0.3,0.5],0.2\rangle,C_{\sigma(4)}=\langle[0.2,0.4],0.1\rangle,$$
$$C_{\sigma(5)}=\langle[0.1,0.3],0.3\rangle.$$

Applying Equation (7) and $w=(0.25,0.15,0.10,0.20,0.30)^T$ is the weighting vector the *COWA* operator such that

$$COWA(c_1,c_2,...,c_n)=\left(\begin{array}{c}[1-\prod_{j=1}^{n}(1-a^{-}_{c_{\sigma(j)}})^{w_j},\ 1-\prod_{j=1}^{n}\left(1-\ a^{+}_{c_{\sigma(j)}})^{w_j}\right)],\\ \prod_{j=1}^{n}\left(\lambda_{c_{\sigma(j)}}\right)^{w_j}\end{array}\right)$$
$$=\left(\begin{array}{c}[1-\prod_{j=1}^{4}(1-a^{-}_{c_{\sigma(j)}})^{w_j},\ 1-\prod_{j=1}^{4}\left(1-\ a^{+}_{c_{\sigma(j)}})^{w_j}\right)],\\ \prod_{j=1}^{n}\left(\lambda_{c_{\sigma(j)}}\right)^{w_j}\end{array}\right)$$
$$=\langle[0.3413,0.5599],0.2810\rangle.$$

The fundamental aspect of the *COWA* operator is the reordering of the arguments to be aggregated, based on their values. The *COWA* operator has different properties which we discuss as follows

Theorem 1. (Commutativity)

$$COWA_w(c_1,c_1,...,c_n)=COWA_w(c_1^{/},c_1^{/},...,c_n^{/})$$

where $(c_1^{/},c_1^{/},...,c_n^{/})$ *is any permutation of* $(c_1,c_1,...,c_n)$.

Proof. Since

$$COWA_w\left(c_1,c_2,...,c_n\right)=w_1c_{\sigma_{(1)}}\oplus w_2c_{\sigma_{(2)}}\oplus...\oplus w_nc_{\sigma_{(n)}}$$
$$COWA_w\left(c_1^{/},c_1^{/},...,c_n^{/}\right)=w_1c^{/}_{\sigma_{(1)}}\oplus w_2c^{/}_{\sigma_{(2)}}\oplus...\oplus w_nc^{/}_{\sigma_{(n)}}$$

since $\left(c_1^{/},c_1^{/},...,c_n^{/}\right)$ is any permutation of $(c_1,c_1,...,c_n)$ then

$$COWA_w\left(c_1,c_1,...,c_n\right)=COWA_w\left(c_1^{/},c_1^{/},...,c_n^{/}\right).$$

Theorem 2. (*Idempotency)*

If $C_j = C$ for all j $(j=1,2,..n)$ where $C=\langle\tilde{A}_c,\lambda_c\rangle$, $=\langle[a_c^-,a_c^+],\lambda_c\rangle$, then

$$COWA_w\left(c_1,c_2,...,c_n\right)=c.$$

Proof. Since $C_j = C\ \forall\ j$, *we have*

$$\begin{aligned}COWA_w\left(c_1,c_2,...,c_n\right)&=w_1c_{\sigma_{(1)}}\oplus w_2c_{\sigma_{(2)}}\oplus...\oplus w_nc_{\sigma_{(n)}}\\&=w_1c\oplus w_2c\oplus...\oplus w_nc\\&=\begin{pmatrix}[1-\prod_{j=1}^{n}(1-\bar{a}_c)^{w_j},\ 1-\prod_{j=1}^{n}\left(1-\ a_c^+)^{w_j}\right)],\\ \prod_{j=1}^{n}(\lambda_c)^{w_j}\end{pmatrix}\\&=\begin{pmatrix}[1-(1-\bar{a}_c)^{\sum_{j=1}^{n}w_j},\ 1-\left(1-\ a_c^+)^{\sum_{j=1}^{n}w_j}\right)],\\ (\lambda_c)^{\sum_{j=1}^{n}w_j}\end{pmatrix}\end{aligned}$$

since $\sum_{j=1}^{n}w_j=1,$ then we can get

$$COWA_w\left(c_1,c_2,...,c_n\right)=\left([\bar{a}_c,a_c^+],\lambda_c\right)=C.$$

Theorem 3. (Monotonicity*)*

If $C_j\leq C_j$ for all *j (j=1,2,...,n)*

$$COWA_w\left(c_1,c_1,...,c_n\right)\leq COWA_w\left(c_1^{/},c_1^{/},...,c_n^{/}\right).$$

Proof. Let

$$COWA_w\left(c_1,c_2,...,c_n\right)=w_1c_{\sigma_{(1)}}\oplus w_2c_{\sigma_{(2)}}\oplus...\oplus w_nc_{\sigma_{(n)}}$$
$$COWA_w\left(c_1^{/},c_1^{/},...,c_n^{/}\right)=w_1c^{/}_{\sigma_{(1)}}\oplus w_2c^{/}_{\sigma_{(2)}}\oplus...\oplus w_nc^{/}_{\sigma_{(n)}}$$

Since $C_j\leq C_j^{/}$ for all *j* it follows that $C_{\sigma(j)}\leq C^{/}_{\sigma(j)}$ where *j (j=1,2,...,n) then*

$$COWA_w\left(c_1,c_1,...,c_n\right)\leq COWA_w\left(c_1^{/},c_1^{/},...,c_n^{/}\right).$$

The CHA Operator

Consider that the *CWA* operator weights only the cubic value set, whereas the *COWA* operator weights only the ordered positions of the *CVs* instead of the weighting the cubic value set themselves. To overcome this limitation, motivated by the idea of combining the *WA* and *OWA* operators, in what follows, we developed a cubic hybrid averaging aggregation *(CHA)* operator, which weights both the given cubic value and its ordered position.

Definition 8. *CHA operator of dimension n is a mapping* $CHA\ :\ \Omega^n\rightarrow\Omega,$ *which has an associated vector* $w=(w_1,w_2,...,w_n)^T$, *with* $w_j\geq 0,$ *j=1,2,...,n, and* $\sum_{j=1}^{n}w_j=1$*, such that*

$$CHA_{w,w}\ (\tilde{c}_1,\tilde{c}_2,,...,\tilde{c}_n)=\left(w_1(c^{\cdot}_{\sigma_{(1)}})\oplus w_2(c^{\cdot}_{\sigma_{(2)}})\oplus...\oplus w_n(c^{\cdot}_{\sigma_{(n)}})\right) \quad (8)$$

where $c^{\cdot}_{\sigma_{(j)}}$ is the jth largest of the weighted CVs $c^{\cdot}_j(c^{\cdot}_j=nw_jc_j,\ j=(1,2,...,n),\ w=(w_1,w_2,...,w_n)^T$ is the weight vector of $c_j\,(j=1,2,...,n)$ with $w_j\geq 0$, and $\sum_{j=1}^{n}w_j=1$, and n is balancing coefficient, which plays a role of balance if the vector $w=(w_1,w_2,...,w_n)^T$ approaches $(\frac{1}{n},\frac{1}{n},...,\frac{1}{n})^T$, then the vector $(nw_1c_1,nw_2c_2,...,nw_nc_n)^T$ approaches $(c_1,c_2,...,c_n)^T$. Let $C^{\cdot}_{\alpha(j)}=\left\langle \bar{A}c^{\cdot}_{\sigma(j)},\dot{\lambda}_{c_{\sigma(j)}}\right\rangle$, then, similar to Theorem 3, we have

$$CHA(c_1,c_2,...,c_n)=w_1c^{\cdot}_{\sigma_{(1)}}\oplus w_2c^{\cdot}_{\sigma_{(2)}}\oplus...\oplus w_nc^{\cdot}_{\sigma_{(n)}}$$

$$=\left(\begin{array}{c}[1-\prod_{j=1}^{n}(1-\dot{a}^{-}_{c_{\sigma(j)}})^{w_j},\ 1-\prod_{j=1}^{n}\left(1-\dot{a}^{+}_{c_{\sigma(j)}})^{w_j}\right)],\\ \prod_{j=1}^{n}\left(\dot{\lambda}_{c_{\sigma(j)}}\right)^{w_j}\end{array}\right) \quad (9)$$

which is called cubic hybrid averaging (*CHA*) operator.

Example 3. *Let* $C_1=\langle[0.3,0.4],0.3\rangle, C_2=\langle[0.5,0.6],0.7\rangle, C_3=\langle[0.4,0.3],0.5\rangle, C_4=\langle[0.6,0.8],0.3\rangle$ *and be four cubic values, and suppose that* $w=(0.20,0.25,0.15,0.40)^T$ *be the weight vector of* C_j *(j=1,2,...,4) , then by operational law* (3) *in Definition* (5), *we get the weighted cubic values such that*

$$c^{\cdot}_1=\langle[0.24,0.33],0.38\rangle,\ c^{\cdot}_2=\langle[0.50,0.60],0.70\rangle$$
$$c^{\cdot}_3=\langle[0.26,0.19],0.65\rangle,\ c^{\cdot}_4=\langle[0.76,0.92],0.14\rangle$$

By applying equation (4), we calculate the scores of C_j *(j=1,2,...,4):*

$$S(c^{\cdot}_1)=\frac{0.24+033-0.38}{3}=0.0600,$$
$$S(c^{\cdot}_2)=\frac{0.50+0.60-0.70}{3}=0.1333$$
$$S(c^{\cdot}_3)=\frac{0.26+0.19-0.65}{3}=-0.0666,$$
$$S(c^{\cdot}_4)=\frac{0.76+0.92-0.14}{3}=0.5133$$

Since $S(c^{\cdot}_4)\geq S(c^{\cdot}_2)\geq S(c^{\cdot}_1)\geq S(c^{\cdot}_3)$ *, hence we can write as follows*

$$c^{\cdot}_{\sigma(1)}=\langle[0.76,0.92],0.14\rangle,\ c^{\cdot}_{\sigma(2)}=\langle[0.50,0.60],0.70\rangle$$
$$c^{\cdot}_{\sigma(3)}=\langle[0.24,0.33],0.38\rangle, c^{\cdot}_{\sigma(4)}=\langle[0.26,0.19],0.65\rangle$$

If the associated weighting vector is $\omega=(0.30,0.10,0.35,0.15)^T$ *, then we aggregate values by using the equation (9) such that*

$$CHA_{w,w}(c_1,c_2,...,c_4)=\left(\begin{array}{c}[1-\prod_{j=1}^{n}(1-\dot{a}^{-}_{c_{\sigma(j)}})^{w_j},\ 1-\prod_{j=1}^{n}\left(1-\dot{a}^{+}_{c_{\sigma(j)}})^{w_j}\right)]\\ ,\prod_{j=1}^{n}\left(\dot{\lambda}_{c_{\sigma(j)}}\right)^{w_j}\end{array}\right)$$

$$=\left(\begin{array}{c}[1-\prod_{j=1}^{4}(1-\dot{a}^{-}_{c_{\sigma(j)}})^{w_j},\ 1-\prod_{j=1}^{4}\left(1-\dot{a}^{+}_{c_{\sigma(j)}})^{w_j}\right)]\\ ,\prod_{j=1}^{4}\left(\dot{\lambda}_{c_{\sigma(j)}}\right)^{w_j}\end{array}\right)$$

$$=\langle[0.4799,0.6580],0.3687\rangle.$$

Theorem 4. The *CWA* operator is a special case of the *CHA* operator.

Proof. Let $w=(\frac{1}{n},\frac{1}{n},...,\frac{1}{n})^T$, then $\dot{C}_j=C_j(j=1,2,...,n)$, so we have

$$GCHA_{w,w}(c_1,c_2,,...,c_n)=\left(w_1(\dot{c}_{\sigma(1)})\oplus w_2(\dot{c}_{\sigma(2)})\oplus...\oplus w_n(\dot{c}_{\sigma(n)})\right)$$
$$=\frac{1}{n}\left((\dot{c}_{\sigma(1)})\oplus(\dot{c}_{\sigma(2)})\oplus...\oplus(\dot{c}_{\sigma(n)})\right)$$
$$=(w_1c_{(1)}\oplus w_2c_{(2)}\oplus...\oplus w_nc_{(n)})$$
$$=CWA_w(c_1,c_2,...,c_n).$$

Theorem 5. *The COWA operator is a special case of the CHA operator.*

Proof. Let $w=(\frac{1}{n},\frac{1}{n},...,\frac{1}{n})^T$, then $\dot{C}_j=C_j(j=1,2,...,n)$, so we have

$$GCHA_{w,w}(c_1,c_2,,...,c_n)=\left(w_1(\dot{c}_{\sigma(1)})\oplus w_2(\dot{c}_{\sigma(2)})\oplus...\oplus w_n(\dot{c}_{\sigma(n)})\right)$$
$$=(w_1(c_{\sigma(1)})\oplus w_2(c_{\sigma(2)})\oplus...\oplus w_n(c_{\sigma(n)}))$$
$$=COWA_w(c_1,c_2,...,c_n).$$

which completes the proof of Theorem.

Approach to Multiple Attribute Group Decision Making with Cubic Information

In this section, we apply the aggregation operator developed to multiple attribute group decision making with cubic information: For this purpose we consider $X=\{x_1,x_2,...,x_m\}$ be a set of m alternatives, $U=\{u_1,u_2,...,u_n\}$ be a set of n attributes, having weighting vector is $w=(w_1,w_2,...,w_n)^T$, $w_j\geq 0$, $j=1,2,...,n$, and $\sum_{j=1}^{n}w_j=1$ and consider $D=\{d_1,d_2,...,d_q\}$ be set of q decision makers, whose weighting vector is $\lambda=\{\lambda_1,\lambda_2,...,\lambda_q\}^T$, where $\lambda_l\in[0,1]$, $l=1,2,...,q$, and $\sum_{l=1}^{q}\lambda_l=1$. Let $A_k=\left(a_{ij}^{(k)}\right)_{m\times n}$ be an cubic decision matrix, where $a_{ij}^{(k)}=\left\langle\tilde{A}_{ij}^{(k)},\lambda_{ij}^{(k)}\right\rangle=\left\langle[\overline{a}_{ij}^{(k)},a_{ij}^{+(k)}],\lambda_{ij}^{(k)}\right\rangle$ is an attribute value provided by decision maker's Dq, denoted by a cubic numbers such that

$$\tilde{A}_{ij}^{(k)}\in[0,1],\ \lambda_{ij}^{(k)}\in[0,1]\qquad i=1,2,...m;\ j=1,2,...,n$$

and construct the cubic decision matrix $R^{(l)}=\left(r_{ij}^{(l)}\right)_{m\times n}$ respectively

Then we apply the *COWA* operator to develop an approach to multiple attribute group decision making with cubic information, which consist of the following steps.

Algorithm

Step 1: In this step, first of all we Construct the individual cubic decision matrix.
Step 2: In this step, we apply the score function and construct the ordered decision matrices.
Step 3: Applying the *COWA* operators of equation (7) to aggregate all the individual cubic decision matrix $R_k=(r_{ij}^{(k)})_{4\times4}(k=1,2,3,4)$ into the collective cubic decision matrix $R=(r_{ij})_{4\times4}$

Step 4: Aggregate all the preference values $r_{ij}=\left\langle\tilde{A}_{ij},\lambda_{ij}\right\rangle$ where *(i=1,2,...,m)* and *(j=1,2,...,n)* by using *COWA* operator to get the complete preference value r_i corresponding to the alternative X_i $(i=1,2,...,m)$

Step 5: In this step, we compute the scores of r_i $(i=1,2,...,m)$. If there is difference between two or more than two score functions then we have must to calculate the accuracy degrees.

Step 6: In this step, we arrange the score values of each alternative by descending order and chose the best alternative by maximum value of score function.

Step 7: End.

Numerical Application

In this section, we will construct a numerical example to show evaluation of the theses with cubic information in order to illustrate the proposed method. Suppose there are four theses, for this purpose let (x_1,x_2,x_3,x_4) are four alternatives such that $X_i(i=1,2,3,4)$ and want to select best one alternative. There are four experts D_q $(q=1,2,3,4)$, from a committee to act the decision makers, having weighting vector $\lambda=(0.20,0.30,0.15,0.35)^T$. Consider there are four attributes (u_1,u_2,u_3,u_4) such that $U_j=(j=1,2,3,4)$.

- u_1 is the language of a thesis;
- u_2 is the innovation;
- u_3 is the rigor;
- u_4 is the structure of of the thesis.

Where $w=(0.1,0.2,0.3,0.4)^T$ is weighting vector of attributes $U_j=(j=1,2,3,4)$. The experts $D_q\ (q=1,2,3,4)$ evaluate the thesis $X_i\ (i=1,2,3,4)$ with respect to attributes such that $U_j=(j=1,2,3,4)$, and develop four cubic decision matrix such that $R_k=(r_{ij}^{(k)})_{4\times4}(k=1,2,3,4)$.

Step 1: The decision makers give his decision in the following tables.

$R^{(1)}=$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.5,0.7],0.5\rangle$	$\langle[0.4,0.6],0.9\rangle$	$\langle[0.3,0.5],0.2\rangle$	$\langle[0.6,0.8],0.3\rangle$
x_2	$\langle[0.3,0.6],0.7\rangle$	$\langle[0.5,0.7],0.8\rangle$	$\langle[0.5,0.6],0.3\rangle$	$\langle[0.3,0.6],0.7\rangle$
x_3	$\langle[0.7,0.8],0.2\rangle$	$\langle[0.4,0.8],0.2\rangle$	$\langle[0.6,0.8],0.5\rangle$	$\langle[0.5,0.4],0.6\rangle$
x_4	$\langle[0.2,0.7],0.6\rangle$	$\langle[0.5,0.9],0.3\rangle$	$\langle[0.4,0.9],0.1\rangle$	$\langle[0.3,0.4],0.5\rangle$

Table 1: Individual cubic decision matrix $R^{(1)}$

$R^{(2)}=$

	u_1	u_2	u_3	u_4
x_1	$\langle 0.3,0.4],0.3\rangle$	$\langle[0.3,0.4],0.2\rangle$	$\langle[0.5,0.7],0.6\rangle$	$\langle[0.4,0.6],0.3\rangle$
x_2	$\langle[0.2,0.6],0.4\rangle$	$\langle[0.8,0.9],0.5\rangle$	$\langle[0.5,0.7],0.5\rangle$	$\langle[0.4,0.8],0.1\rangle$
x_3	$\langle[0.6,0.7],0.2\rangle$	$\langle[0.3,0.4],0.4\rangle$	$\langle[0.5,0.7],0.4\rangle$	$\langle[0.3,0.5],0.4\rangle$
x_4	$\langle[0.6,0.8],0.1\rangle$	$\langle[0.4,0.5],0.7\rangle$	$\langle[0.5,0.9],0.3\rangle$	$\langle[0.3,0.4],0.2\rangle$

Table 2: Individual cubic decision matrix $R^{(2)}$

$R^{(3)}=$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.2,0.5],0.3\rangle$	$\langle[0.3,0.6],0.1\rangle$	$\langle[0.3,0.4],0.3\rangle$	$\langle[0.4,0.5],0.3\rangle$
x_2	$\langle[0.1,0.4],0.1\rangle$	$\langle[0.3,0.8],0.8\rangle$	$\langle[0.2,0.3],0.3\rangle$	$\langle[0.2,0.5],0.6\rangle$
x_3	$\langle[0.4,0.7],0.3\rangle$	$\langle[0.5,0.7],0.6\rangle$	$\langle[0.4,0.9],0.3\rangle$	$\langle[0.3,0.5],0.3\rangle$
x_4	$\langle[0.3,0.5],0.3\rangle$	$\langle[0.3,0.5],0.5\rangle$	$\langle[0.3,0.7],0.4\rangle$	$\langle[0.4,0.5],0.2\rangle$

Table 3: Individual cubic decision matrix $R^{(3)}$

$R^{(4)}=$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.4,0.6],0.9\rangle$	$\langle[0.2,0.5],0.5\rangle$	$\langle[0.2,0.5],0.4\rangle$	$\langle[0.4,0.5],0.3\rangle$
x_2	$\langle[0.5,0.8],0.3\rangle$	$\langle[0.2,0.4],0.5\rangle$	$\langle[0.4,0.5],0.7\rangle$	$\langle[0.3,0.6],0.2\rangle$
x_3	$\langle[0.2,0.3],0.4\rangle$	$\langle[0.1,0.5],0.2\rangle$	$\langle[0.3,0.9],0.5\rangle$	$\langle[0.3,0.5],0.3\rangle$
x_4	$\langle[0.5,0.5],0.3\rangle$	$\langle[0.3,0.5],0.5\rangle$	$\langle[0.3,0.7],0.4\rangle$	$\langle[0.4,0.5],0.5\rangle$

Table 4: Individual cubic decision matrix $R^{(4)}$

Step 2: In this step we applying equation (4) and find the ordered of the ordered of Individual cubic decision matrix.

$R^{(1)}=$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.6,0.8],0.3\rangle$	$\langle[0.5,0.7],0.5\rangle$	$\langle[0.3,0.5],0.2\rangle$	$\langle[0.4,0.6],0.9\rangle$
x_2	$\langle[0.5,0.6],0.3\rangle$	$\langle[0.5,0.7],0.8\rangle$	$\langle[0.3,0.6],0.7\rangle$	$\langle[0.3,0.6],0.7\rangle$
x_3	$\langle[0.7,0.8],0.2\rangle$	$\langle[0.4,0.8],0.2\rangle$	$\langle[0.6,0.8],0.5\rangle$	$\langle[0.5,0.4],0.6\rangle$
x_4	$\langle[0.4,0.9],0.1\rangle$	$\langle[0.5,0.9],0.3\rangle$	$\langle[0.2,0.7],0.6\rangle$	$\langle[0.4,0.9],0.1\rangle$

Table 5: Individual cubic ordered decision matrix $R^{(1)}$

$R^{(2)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.4,0.6],0.3\rangle$	$\langle[0.5,0.7],0.6\rangle$	$\langle[0.3,0.4],0.2\rangle$	$\langle0.3,0.4],0.3\rangle$
x_2	$\langle[0.8,0.9],0.5\rangle$	$\langle[0.4,0.8],0.1\rangle$	$\langle[0.5,0.7],0.5\rangle$	$\langle[0.2,0.6],0.4\rangle$
x_3	$\langle[0.6,0.7],0.2\rangle$	$\langle[0.5,0.7],0.4\rangle$	$\langle[0.3,0.5],0.4\rangle$	$\langle[0.3,0.4],0.4\rangle$
x_4	$\langle[0.6,0.8],0.1\rangle$	$\langle[0.3,0.4],0.2\rangle$	$\langle[0.1,0.4],0.4\rangle$	$\langle[0.3,0.4],0.2\rangle$

Table 6: Individual cubic ordered decision matrix $R^{(2)}$

$R^{(3)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.3,0.6],0.1\rangle$	$\langle[0.4,0.5],0.3\rangle$	$\langle[0.2,0.5],0.3\rangle$	$\langle[0.3,0.4],0.3\rangle$
x_2	$\langle[0.1,0.4],0.1\rangle$	$\langle[0.3,0.8],0.8\rangle$	$\langle[0.2,0.3],0.3\rangle$	$\langle[0.2,0.5],0.6\rangle$
x_3	$\langle[0.4,0.9],0.3\rangle$	$\langle[0.4,0.7],0.3\rangle$	$\langle[0.5,0.7],0.6\rangle$	$\langle[0.3,0.5],0.3\rangle$
x_4	$\langle[0.4,0.5],0.2\rangle$	$\langle[0.3,0.7],0.4\rangle$	$\langle[0.3,0.5],0.3\rangle$	$\langle[0.3,0.5],0.5\rangle$

Table 7: Individual cubic ordered decision matrix $R^{(3)}$

$R^{(4)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle[0.4,0.6],0.9\rangle$	$\langle[0.4,0.5],0.3\rangle$	$\langle[0.2,0.5],0.4\rangle$	$\langle[0.2,0.5],0.5\rangle$
x_2	$\langle[0.5,0.8],0.3\rangle$	$\langle[0.3,0.6],0.2\rangle$	$\langle[0.4,0.5],0.7\rangle$	$\langle[0.2,0.4],0.5\rangle$
x_3	$\langle[0.3,0.9],0.5\rangle$	$\langle[0.3,0.5],0.3\rangle$	$\langle[0.1,0.5],0.2\rangle$	$\langle[0.2,0.3],0.4\rangle$
x_4	$\langle[0.5,0.5],0.3\rangle$	$\langle[0.3,0.7],0.4\rangle$	$\langle[0.4,0.5],0.5\rangle$	$\langle[0.3,0.5],0.5\rangle$

Table 8: Individual cubic ordered decision matrix $R^{(4)}$

Step 3: Using the *COWA* operator to aggregate all the different cubic ordered decision matrices, into the single matrix.

	u_1	u_2	u_3	u_4
x_1	$\langle[0.39,0.62],0.33\rangle$	$\langle[0.43,0.57],0.36\rangle$	$\langle[0.23,0.51],0.29\rangle$	$\langle[0.27,0.46],0.41\rangle$
x_2	$\langle[0.50,0.59],0.23\rangle$	$\langle[0.34,0.72],0.30\rangle$	$\langle[0.35,0.51],0.50\rangle$	$\langle[0.40,0.49],0.52\rangle$
x_3	$\langle[0.45,0.86],0.32\rangle$	$\langle[0.38,0.64],0.30\rangle$	$\langle[0.33,0.60],0.35\rangle$	$\langle[0.28,0.39],0.38\rangle$
x_4	$\langle[0.48,0.69],0.19\rangle$	$\langle[0.32,0.69],0.33\rangle$	$\langle[0.29,0.50],0.41\rangle$	$\langle[0.31,0.49],0.35\rangle$

Table 9: Collective cubic ordered decision matrix

Step 4: Now in this step we aggregate all the preference values $r_{ij} = \langle \tilde{A}_{ij}, \lambda_{ij} \rangle$ where $(i = 1,2,...m;\ j = 1,2,...,n)$ by using *COWA* operator to get the complete preference value r_i corresponding to the alternative X_i $(i=1,2,...,m)$

$$(r_1) = \langle[0.3408, 0.5366], 0.3584\rangle, (r_2) = \langle[0.3975, 0.5946], 0.3723\rangle$$
$$(r_3) = \langle[0.3546, 0.6358], 0.3378\rangle, (r_4) = \langle[0.3480, 0.6035], 0.3116\rangle$$

Step 5: In this step we compute the scores of r_i $(i=1,2,3,4)$

$$S(r_1) = 0.1730, S(r_2) = 0.2066, S(r_3) = 0.2175, S(r_4) = 0.2133.$$

Step 6: Now we arrange the score values of each alternative by descending order and chose the best alternative by maximum value of score function $S(r_3) > S(r_4) > S(r_2) > S(r_1)$. Thus $x_3 > x_4 > x_2 > x_1$, hence x_3 is best alternative.

Step 7: End.

Further Discussion

In order to show the validity and effectiveness of the proposed methods, we utilize intuitionisti fuzzy *(IFS)* sets to solve the same problem described above. We apply the proposed aggregation operators developed in this paper. After simplification we can get the ranking result as $x_3 > x_4 > x_2 > x_1$, we find that x_3 is best alternative. In the above example, if we use *IFS* sets to express the decision maker's evaluations then the decision matrix $R^{(1)},R^{(2)},R^{(3)},R^{(4)}$ can be written as decision matrix $R^{(11)},R^{(22)},R^{(33)},R^{(44)}$ deleting the cubic numbers which are shown respectively. In [20], the proposed *IFWA* operators to deal with multiple attribute decision making with intuitionistic fuzzy information respectively such that,

$R^{(11)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle 0.5,,0.5\rangle$	$\langle 0.4,0.9\rangle$	$\langle 0.3,0.2\rangle$	$\langle 0.6,0.3\rangle$
x_2	$\langle 0.3,0.7\rangle$	$\langle 0.5,0.8\rangle$	$\langle 0.5,0.3\rangle$	$\langle 0.3,0.7\rangle$
x_3	$\langle 0.7,0.2\rangle$	$\langle 0.4,0.2\rangle$	$\langle [0.6,0.5\rangle$	$\langle 0.5,0.6\rangle$
x_4	$\langle 0.2,0.6\rangle$	$\langle 0.5,0.3\rangle$	$\langle 0.4,0.1\rangle$	$\langle 0.3,0.5\rangle$

$R^{(22)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle 0.3,0.3\rangle$	$\langle 0.3,0.2\rangle$	$\langle 0.5,0.6\rangle$	$\langle 0.4,0.3\rangle$
x_2	$\langle 0.2,0.4\rangle$	$\langle 0.8,0.5\rangle$	$\langle 0.5,0.5\rangle$	$\langle 0.4,0.1\rangle$
x_3	$\langle 0.6,0.2\rangle$	$\langle 0.3,0.4\rangle$	$\langle 0.5,0.4\rangle$	$\langle 0.3,0.4\rangle$
x_4	$\langle 0.6,0.1\rangle$	$\langle 0.4,0.7\rangle$	$\langle 0.5,0.3\rangle$	$\langle 0.3,0.2\rangle$

$R^{(33)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle 0.2,0.3\rangle$	$\langle 0.3,0.1\rangle$	$\langle 0.3,0.3\rangle$	$\langle 0.4,0.3\rangle$
x_2	$\langle 0.1,0.1\rangle$	$\langle 0.3,0.8\rangle$	$\langle 0.2,0.3\rangle$	$\langle 0.2,0.6\rangle$
x_3	$\langle 0.4,0.3\rangle$	$\langle 0.5,0.6\rangle$	$\langle 0.4,0.3\rangle$	$\langle 0.3,0.3\rangle$
x_4	$\langle 0.3,0.3\rangle$	$\langle 0.3,0.5\rangle$	$\langle 0.3,0.4\rangle$	$\langle 0.4,0.2\rangle$

$R^{(44)} =$

	u_1	u_2	u_3	u_4
x_1	$\langle 0.4,0.9\rangle$	$\langle 0.2,0.5\rangle$	$\langle 0.2,0.4\rangle$	$\langle 0.4,0.3\rangle$
x_2	$\langle 0.5,0.3\rangle$	$\langle 0.2,0.5\rangle$	$\langle 0.4,0.7\rangle$	$\langle 0.3,0.2\rangle$
x_3	$\langle 0.2,0.4\rangle$	$\langle 0.1,0.2\rangle$	$\langle 0.3,0.5\rangle$	$\langle 0.3,0.3\rangle$
x_4	$\langle 0.5,0.3\rangle$	$\langle 0.3,0.5\rangle$	$\langle 0.3,0.4\rangle$	$\langle 0.4,0.5\rangle$

We further explain to find the best alternative of *IFS* , after the computation process of the overall preference values are as follows. By applying score function of such that,

$$S(r_1) = -0.0176, S(r_2) = 0.0252, S(r_3) = 0.0168, S(r_4) = 0.0364.$$

Now we find the ranking as $x_4 > x_2 > x_3 > x_1$ In this case, x_4 is the best alternative. It is noted that the ranking orders obtained by this paper and by [20] are very different. Therefore, *CFNs* may better reflect the decision information than *IFNs*. Hence our proposed approach is better than *IFNs*.

Conclusion

In this paper, we have investigated the multiple attribute group decision making problems in which attribute values are in the form of cubic numbers. We first define some operations of *CNs* and some corresponding operational laws. Further, we have proposed some new aggregation operators for *CNs*, including cubic weighted averaging *(CWA)* operator, cubic ordered weighted averaging *(COWA)* operator and cubic hybrid averaging *(CHA)* operator. And desirable properties of the operators have also been analyzed. Finally, an illustrative example has been constructed to show the proposed *MAGDB* method. Our proposed method is different from all the previous techniques for group decision making due to the fact that the proposed method use cubic fuzzy information, which will not cause any loss of information in the process. So it efficient and feasible for real-world decision making applications.

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