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Picture Fuzzy Grey Approach for Decision Problems with unknown Weight Information

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Abstract

Cuong's picture fuzzy set (PFS) has more capability to grip the uncertainties in real-life decision-making problems as compare to intuitionistic fuzzy set. The purpose of this paper is to introduce a new Grey approach in which the attribute values takes from the picture fuzzy numbers, attribute weights information is unknown and develop a multi criteria decision making approach to study the interaction between the input argument under the picture fuzzy environment. The main advantage of the proposed technique is that, it can deal with the situations of the positive interaction, negative interaction or non-interaction among the criteria, during decision-making process. Finally, a numerical approach is demonstrated for implementation of proposed technique and show that how proposed technique is reliable and effective is illustrated.

Keywords: Picture Fuzzy Set; Averaging Aggregation Operator; Geometric Aggregation Operator; Normalized Hamming Distance; Grey Approach

Introduction

In the life of human beings' decision making is a worldwide procedure, which can be designated as final consequence of certain spiritual and intellectual procedures that commanding to variety of the finest alternative. Based on fuzziness circumstances; assigned membership grades to elements of a set in the interval [0,1] by offering the idea of fuzzy set (FS). Zadeh's work in this direction is remarkable as many of set theoretic properties of crisp cases were defined for fuzzy sets. In FSs each element "x"of the domain set contains only one index namely as degree of membership "$P(x)$" which oscillates from 0 to 1. Non-membership degree for the FS is straightforward equivalent to "$1-P(x)$". FSs got the attentions of researchers and found its applications in decision science, intelligence science, communications, engineering, computer sciences etc [1].

Form last few decades, the intuitionistic fuzzy set; fruitful and broadly utilized by the researchers to grip the ambiguity and imprecision data [2]. To cumulate all the executive of criteria for alternatives, aggregation operators, play vital character throughout the information merging procedure. Xu Z, presented weighted averaging operator while developed geometric aggregation operator for aggregating the different intuitionistic fuzzy numbers [3,4]. In some decision theories, the decision makers deal with the situation of particular attributes where values of their summation of membership degrees exceed 1. In such condition, IFSs has no ability to obtain any satisfactory result. To overcome this situation Yager RR developed the idea of Pythagorean fuzzy set as a generalization of IFS, which satisfies that the value of square summation of its membership degrees is less then or equals to 1. In particular situation where the neutral membership degree calculates independently in real life problems [5]. In such condition, Pythagorean fuzzy set fails to attain any satisfactory result. Based on these circumstances, to overcome this situation, Cuong BC and Kreinovitch V, initiated the idea of picture fuzzy set (PFSs) [6]. They utilized three indexes (membership degree"$P(x)$", neutral-membership degree "$I(x)$"and non-membership degree "$N(x)$") in PFS with axiom that is $0 \le P(x)+I(x)+N(x) \le 1$.

PFS is the generalization of FS and IFS. Obviously PFSs are more suitable than IFS to deal with fuzziness and vagueness. Garg H, introduced picture fuzzy weighted averaging operator, Picture fuzzy ordered weighted averaging operator and Picture fuzzy hybrid averaging operator under picture fuzzy environment [7]. In Singh P, introduced a correlation coefficient for the PFS [8]. Wei G, established decision making technique depending on the picture fuzzy weighted cross-entropy and used to differentiate the alternatives [9]. In Ashraf et al., investigated the multiple attribute decision problems based on the picture fuzzy setting, also developed some picture fuzzy geometric operators and discussed their basic properties [10]. In Zeng et al., proposed the application of exponential Jensen picture fuzzy divergence measure in multi-criteria group decision making problems [11]. In Khan et al., proposed the concept of generalized picture fuzzy soft set and discussed their applications in decision making problems. In Ashraf et al., proposed the concept of cubic picture fuzzy set and discussed their applications [13]. In Ashraf et al., proposed the concept of picture fuzzy linguistic fuzzy set and discussed their applications in decision making problems [14].

Sometime, DM utilizes the picture fuzzy information and due to lack of knowledge about domain of problem and time limiting stress, the knowledge about weights is unknown. Existing intuitionistic fuzzy grey method will flop to handle picture fuzzy information where information about weights is unknown [15,16]. Utilizing incomplete weight information of the alternatives as well as the picture fuzzy information to construct weights of the alternative to use grey approach which is a finest and motivational research issue. Therefore, this work upgrades the idea of grey to establish new methodology for resolving decision problems under picture fuzzy setting, in which we assumed incomplete known information about weights of alternative.

The objectives of this paper are: (1) to discuss the picture fuzzy number (PFNs) and related basic operational identities, (2) to suggest score and accuracy functions for comparison, (4) to propose picture aggregation operators and some debate on their properties, (5) to demonstrate a MADM method based on to traditional grey method with incomplete weight information under picture fuzzy setting,

The superfluity of this paper is planned as follows. Section "Preliminaries" gives brief reassess the initial ideas related to IFSs and PFSs and their properties. In next sections "Comparison Rules for PFNs" and "Picture Fuzzy Number Aggregation Operators" define a rule which utilized to rank the alternatives and then present the aggregated operators. In sections "Grey Relation for Decision Making Based on Picture Fuzzy Setting", MADM method is proposed to deal with picture fuzzy setting and in last descriptive examples to express the effectiveness and reliability of the suggested technique, is illustrated.

Preliminaries

The paper gives brief discussion on basic ideas associated to IFS and PFS with their operations and operators. Also discuss more familiarized ideas which utilized in following analysis.

Definition 1: Let the universe set R. Then A is known to be an IFS of R, if

$$A = \{< \alpha, P_A(\alpha), N_A(\alpha) >| \alpha \in R\},$$

where $P_A : R \to [0,1]$ and $N_A : R \to [0,1]$ are said to be positive-membership degree of α in R and negative-membership degree of α in R respectively. Also P_A and N_A fulfil the following condition,

$$(0 \le P_A(\alpha) + N_A(\alpha) \le 1), (\forall \alpha \in R) \qquad (1)$$

Definition 2: Let the universe set R. Then A is known to be PFS of R, if

$$A = \{< \alpha, P_A(\alpha), I_A(\alpha), N_A(\alpha) >| \alpha \in R\},$$

where $P_A : A \to [0,1]$, $I_A : A \to [0,1]$ and $N_A : A \to [0,1]$ are said to be degree of positive membership of α in R, neutral membership degree of α in R and negative-membership degree of α in R respectively [6]. Also P_A, I_A and N_A satisfy the following condition:

$$\text{ü§üüüR} \qquad \le P_A \;\alpha\; + I_A \;\alpha\; + N_A \;\alpha\; \le$$

and $\gamma_A(\alpha) = 1 - (P_A(\alpha) + I_A(\alpha) + N_A(\alpha))$ said to be refusal membership degree of α in R For PFS $\langle (P_A(\alpha), I_A(\alpha), N_A(\alpha)) \rangle$ are said to picture fuzzy number (PFN) and each PFN can be denoted by $e = \langle P_e, I_e, N_e \rangle$, where P_e, I_e and $N_e \in [0,1]$, with condition that $0 \le P_e + I_e + N_e \le 1$.

Definition 3: Let $e_j = \left\langle P_{e_j}, I_{e_j}, N_{e_j} \right\rangle$ and $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$ be two PFNs, and $\mu > 0$ [6]. Then the operations for PFNs are defined as:

(1) $$e_j \oplus e_k = \left\langle P_{e_j} + P_{e_k} - P_{e_j} \cdot P_{e_k},\ I_{e_j} \cdot I_{e_k},\ N_{e_j} \cdot N_{e_k} \right\rangle$$

(2) $$\mu e_j = \left\langle 1 - (1 - P_{e_j})^{\mu},\ (I_{e_j})^{\mu},\ (N_{e_j})^{\mu} \right\rangle$$

(3) $$e_j \otimes e_k = \left\langle P_{ej} \cdot P_{e_{\text{üüü}}},\ I_e \cdot I_e,\ N_e + N_e - N_e \cdot N_e \right\rangle;$$

(4) $$e_j^{\mu} = \left\langle (P_{e_j})^{\mu},\ (I_{e_j})^{\mu},\ 1-(1-N_{e_j})^{\mu} \right\rangle.$$

Definition 4: Let the universe set $R = \{r_1$ üẅü $r_n\}$ and $e_j = \left\langle P_{e_j}, I_{e_j}, N_{e_j} \right\rangle$ & $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$ be any two PFNs [6]. Then

$d(e_i, e_j) = \frac{1}{n}\sum_{i=1}^{n}\left(\left|P_{e_i}(a_i) - P_{e_j}(a_i)\right| + \left|I_{e_i}(a_i) - I_{e_j}(a_i)\right| + \left|N_{e_i}(a_i) - N_{e_j}(a_i)\right|\right)$ is known to be Normalized Hamming distance between two PFNs e_j & e_k.

Comparison Rules

Here we define some functions like score function and accuracy function [3]. Which play important role for ranking of PFNs is described as:

Definition 5: Let $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$ $k \in N$ be the PFNs [10]. Then

(1) $$sc(e_k) = (P_{e_k} - N_{e_k});$$

(2) $$ac(e_k) = P_{e_k} + I_{e_k} + N_{e_k};$$

where sc(e_k) and ac(e_k) are said to be score and accuracy functions of the PFNs, the technique for comparison of PFNs can be described in next definition as.

Definition 6: Let $e_j = \left\langle P_{e_j}, I_{e_j}, N_{e_j} \right\rangle$ and $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$ be any two PFNs [10]. Then by using the Definition 5, comparison technique can be described as,

(a) If sc(e_j) = sc(e_k), then $e_j > e_k$

(b) If sc(e_j) = sc(e_k), and ac(e_j) > ac(e_k) then $e_j > e_k$

(c) If sc(e_j) = sc(e_k), and ac(e_j) = ac(e_k) then $e_j = e_k$

Definition 7: Let $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$, $(k \in N)$ be any collection of PFNs [7]. Then picture fuzzy number weighted averaging aggregat ing (PFNWAA) operator, $PFNWAA : PFN^n \to PFN$, describe as,

$$PFNWAA(e_1 \text{üẅü}\ e_n) = \sum_{k=1}^{n} \tau_k e_k$$

in which $\tau = \{\tau_1, \tau_2, ..., \tau_n\}$ be the weight vector of $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$, $k \in N$, with $\tau_k \geq 0$ and $\sum_{k=1}^{n} \tau_k = 1.$

Definition 8: Let $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$ $k \in N$ be a collection of PFNs [7]. Then the picture fuzzy number order weighted averaging aggregating (PFNOWAA) operator, $PFNOWAA : PFN^n \to PFN$, describe as,

$$PFNOWAA_w(e_1, e_2, ..., e_n) = \Pi_{k=1}^{n} \tau_k e_{\eta(k)}$$

with dimensions *n*, where kth biggest weighted value is $e_{\eta(k)}$ consequently by total order $e_{\eta(1)} \geq e_{\eta(2)} \geq ... \geq e_{\eta(n)}$. $\tau = \{\tau_1, \tau_2, ..., \tau_n\}$ are the weight vectors such that $\tau_k \geq 0$ $(k \varepsilon N)$ and $\sum_{k=1}^{n} \tau_k = 1$.

Definition 9: Let $e_k = \left\langle P_{e_k}, I_{e_k}, N_{e_k} \right\rangle$ $k \in N$ be a collection of PFNs. Then the picture fuzzy number hybrid weighted averaging aggregating (PFNHWAA) operator, $PFNHWAA : PFN^n \to PFN$, describe as,

$$PFNHWAA_w(e_1, e_2, ...e_n) = \Pi_{k=1}^{n} \tau_k e'_{\eta(k)},$$

With dimensions *n*, where kth biggest weighted value is $e_{\eta(k)}$ and $e'_k\left(e'_k = n\tau_k e_k,\ k\varepsilon N\right)$, $\tau = \{\tau_1, \tau_2, ..., \tau_n\}$ are the weight vectors such that $\tau_k \geq 0 \quad k \in N$ and $\sum_{k=1}^{n} \tau_k = 1$. Also $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the associated weights such that $\omega_k \geq 0 \quad k \in N$ and $\sum_{k=1}^{n} \omega_k = 1$, and balancing coefficient is n [7].

Definition 10: Let $e_k = \langle P_{e_k}, I_{e_k}, N_{e_k} \rangle, \ k \in N$ be any collection of PFNs [10]. Then the picture fuzzy number weighted geometric averaging (PFNWGA) aggregated operator, $PFNWGA : PFN^n \rightarrow PFN$, describe as,

$$PFNWGA(e_1, e_2, \ldots, e_n) = \prod_{k=1}^{n} e_k^{\tau_k}$$

In which $\tau = \{\tau_1, \tau_2, ..., \tau_n\}$ be the weight vector of $e_k = \langle P_{e_k}, I_{e_k}, N_{e_k} \rangle$, , $k \in N$ with $\tau_k \geq 0$ and $\sum_{k=1}^{n} \tau_k = 1.$

Definition 11: Let $e_k = \langle P_{e_k}, I_{e_k}, N_{e_k} \rangle, \ k \in N$ be any collection of PFNs [10]. Then picture fuzzy number order weighted geometric averaging (PFNOWGA) operator, $PFNOWGA : PFN^n \rightarrow PFN$, describe as,

$$PFNOWGA_w(e_1, e_2, ..., e_n) = \Pi_{k=1}^{n} e_{\eta(k)}^{\tau_k},$$

with dimensions n, where *kth* biggest weighted value is $e_{\eta(k)}$ consequently by total order $e_{\eta(1)} \geq e_{\eta(2)} \geq ... \geq e_{\eta(n)}$. $\tau = \{\tau_1, \tau_2, ..., \tau_n\}$ be the weight vector of $e_k = \langle P_{e_k}, I_{e_k}, N_{e_k} \rangle$, , $k \in N$ with $\tau_k \geq 0$ and $\sum_{k=1}^{n} \tau_k = 1.$

Definition 12: Let $e_k = \langle P_{e_k}, I_{e_k}, N_{e_k} \rangle, \ k \in N$ be any collection of PFNs. Then the picture fuzzy number hybrid weighted geometric averaging (PFNHWGA) operator, $PFNHWGA : PFN^n \rightarrow PFN$, describe as,

$$PFNHWGA_w(e_1, e_2, ...e_n) = \Pi_{j=1}^{n} e_{\eta(k)}^{'\tau_k},$$

with dimensions n, where *kth* biggest weighted value is $e_{\eta(k)}$ and $e_k^{'} \left(e_k^{'} = n\tau_k e_k, \ k\varepsilon N\right), \ \tau = \{\tau_1, \tau_2, ..., \tau_n\}$ be the weight vector of $e_k = \langle P_{e_k}, I_{e_k}, N_{e_k} \rangle, \ k \in N$, with $\tau_k \geq 0$ and $\sum_{k=1}^{n} \tau_k = 1.$ Also $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the associated weights such that $\omega_j \geq 0$ and $(j\varepsilon N)$, and $\sum_{j=1}^{n} \omega_j = 1$ balancing coefficient is n [10].

Grey Approach for Decision Problems Based on Picture Fuzzy Setting

Suppose that $A = \{a_1, a_2, ..., a_n\}$, n alternatives and $C = \{c_1, c_2, ..., c_m\}$, m criteria and criteria of weight vector are $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_m)$, where $\vartheta_k \geq 0 \ (k = 1, 2, ..., n), \ \sum_{k=1}^{n} \vartheta_k = 1$. Suppose that DM deliver information about weights of criteria may be denotes in the following form [17] for $j \neq k$,

(1) If $\{\vartheta_j \geq \vartheta_k\}$ then ranking is weak.

(2) If $\{\vartheta_j - \vartheta_k \geq \lambda_j (> 0)\}$, then ranking is strict.

(3) If $\{\vartheta_j \geq \lambda_j \vartheta_k\}, 0 \leq \lambda_j \leq 1$, then ranking is multiple ranking.

(4) If $\{\aleph\aleph\aleph \leq \quad + \quad \}, 0 \leq \quad \leq \quad + \quad \leq 1$, then ranking is an interval ranking.

For convenience, we denote by Δ; the set of information about weights of alternative are known which are provided by the experts.

Assuming that there are decision makers $D = \{d_1, d_2, ..., d_k\}$, whose corresponding weight vector is $\mu = (\mu_1, \mu_2, ..., \mu_k)$. Grey method under picture fuzzy information's are described with these following steps.

Let $e = (e_{jk}^{i})_{n \times m}$ be the picture fuzzy decision matrix,

$$
\begin{array}{cccc}
\left\langle P_{e^i}, I_{e^i}, N_{e^i} \right\rangle & \left\langle P_{e^i}, I_{e^i}, N_{e^i} \right\rangle & \ldots & \left\langle P_{e^i{}_m}, I_{e^i{}_m}, N_{e^i{}_m} \right\rangle \\
\left\langle P_{e^i}, I_{e^i}, N_{e^i} \right\rangle & \left\langle P_{e^i}, I_{e^i}, N_{e^i} \right\rangle & \ldots & \left\langle P_{e^i{}_m}, I_{e^i{}_m}, N_{e^i{}_m} \right\rangle \\
. & . & . & . \\
. & . & . & . \\
. & . & . & . \\
\left\langle P_{e^i_{n1}}, I_{e^i_{n1}}, N_{e^i_{n1}} \right\rangle & \left\langle P_{e^i_{n2}}, I_{e^i_{n2}}, N_{e^i_{n2}} \right\rangle & \ldots & \left\langle P_{e^i_{nm}}, I_{e^i_{nm}}, N_{e^i_{nm}} \right\rangle
\end{array}
$$

Where $\left\langle P_{e^i_{jk}}, I_{e^i_{jk}}, N_{e^i_{jk}} \right\rangle$ is a PFN representing the performance rating of the alternative $a_j \in A$ with respect to the attribute $A_j \in A$ provided by the decision makers D.

Step: 1

Weight the attribute value of each alternative by utilizing the operations of PFSs.

Step: 2

Calculate the score function by utilizing the definition 6.

Step: 3

The picture fuzzy positive-ideal solution (PFPIS), denoted by $P^+ = \left\{P_1^+, P_2^+, \ldots, P_n^+\right\}$ and the picture fuzzy negative-ideal solution (PFNIS), denoted by $P^- = \left\{P_1^-, P_2^-, \ldots, P_n^-\right\}$ are defined as

$P_k^+ = \max_k sc_{lk}$ and $P_k^- = \min_k sc_{lk}$.

Step: 4

According to Picture fuzzy distance, calculate the distance between the alternative A_i and the PFPIS P^+ and the distance between the alternative A_i and the PFNIS P^-, respectively:

$$d(e_j, e_k) = \tfrac{1}{n}\sum_{j=1}^{n}\left(\left|P_{e_j}\left(a_j\right) - P_{e_k}\left(a_j\right)\right| + \left|I_{e_j}\left(a_j\right) - I_{e_k}\left(a_j\right)\right| + \left|N_{e_j}\left(a_j\right) - N_{e_k}\left(a_j\right)\right|\right)$$

This distance is known to be Normalized Hamming distance d (e_j, e_k), and construct a Picture fuzzy positive-ideal separation matrix D^+ and Picture fuzzy negative-ideal separation matrix D^- as follows:

$$D^+ = (D_{jk}^+)_{n\times m} = \begin{bmatrix} d\left(P_{e_{11}}, P_1^+\right) & d\left(P_{e_{12}}, P_2^+\right) & \ldots & d\left(P_{e_{1m}}, P_m^+\right) \\ d\left(P_{e_{21}}, P_1^+\right) & d\left(P_{e_{22}}, P_2^+\right) & \ldots & d\left(P_{e_{2m}}, P_m^+\right) \\ . & . & & . \\ . & . & \ldots & . \\ . & . & & . \\ d\left(P_{e_{n1}}, P_1^+\right) & d\left(P_{e_{n2}}, P_2^+\right) & \ldots & d\left(P_{e_{nm}}, P_m^+\right) \end{bmatrix}$$

and

$$D^- = (D_{jk}^-)_{n\times m} = \begin{bmatrix} d\left(P_{e_{11}}, P_1^-\right) & d\left(P_{e_{12}}, P_2^-\right) & \ldots & d\left(P_{e_{1m}}, P_m^-\right) \\ d\left(P_{e_{21}}, P_1^-\right) & d\left(P_{e_{22}}, P_2^-\right) & \ldots & d\left(P_{e_{2m}}, P_m^-\right) \\ . & . & & . \\ . & . & \ldots & . \\ . & . & & . \\ d\left(P_{e_{n1}}, P_1^-\right) & d\left(P_{e_{n2}}, P_2^-\right) & \ldots & d\left(P_{e_{nm}}, P_m^-\right) \end{bmatrix}$$

Step: 5

Grey coefficient for each alternative calculated from PIS and NIS by utilizing following below equation.

The grey coefficient for each alternative calculated from PIS is provided as:

$$\xi_{jk}^{+} = \frac{\min_{1\le j\le m}\min_{1\le k\le n} d\left(P_{e_{jk}}, P_k^{+}\right) + \rho \max_{1\le j\le m}\max_{1\le k\le n} d\left(P_{e_{jk}}, P_k^{+}\right)}{d\left(P_{e_{jk}}, P_k^{+}\right) + \rho \max_{1\le j\le m}\max_{1\le k\le n} d\left(P_{e_{jk}}, P_k^{+}\right)}$$

Where $j = 1,2,3,....,m$ and $k = 1,2,3,....,n$.

Similarly, the grey coefficient of each alternative calculated from NIS is provided as

$$\xi_{jk}^{-} = \frac{\min_{1\le j\le m}\min_{1\le k\le n} d\left(P_{e_{jk}}, P_k^{-}\right) + \rho \max_{1\le j\le m}\max_{1\le k\le n} d\left(P_{e_{jk}}, P_k^{-}\right)}{d\left(P_{e_{jk}}, P_k^{-}\right) + \rho \max_{1\le j\le m}\max_{1\le k\le n} d\left(P_{e_{jk}}, P_k^{-}\right)}$$

Where $j = 1,2,3,....,m$ and $k = 1,2,3,....,n$ the identification coefficient $\rho = 0.5$

Step: 6

Calculating the grey coefficient degree for each alternative from PIS and NIS by utilizing following below equation, respectively,

$$\xi_j^{+} = \sum_{k=1}^{n} \vartheta_k \xi_{jk}^{+}$$

$$\xi_j^{-} = \sum_{k=1}^{n} \vartheta_k \xi_{jk}^{-}$$

The basic principle of the Grey method is that the chosen alternative should have the largest degree of grey relation from the PIS and the smallest degree of grey relation from the NIS. Obviously, for the weights are known, the smaller ξ_j^- and the larger ξ_j^+, the finest alternative A_j. But incomplete information about weights of alternatives is known. So, in this circumstance the ξ_j^- and ξ_j^+ information about weight calculated initially. So, we provide following optimization models for multiple objectives to calculate the information about weight,ξ

$$(OM1)\begin{cases} \min \xi_j^{-} = \sum_{k=1}^{n} \vartheta_k \xi_{jk}^{-} & j = 1,2,...,m \\ \max \xi_j^{+} = \sum_{k=1}^{n} \vartheta_k \xi_{jk}^{+} & j = 1,2,...,m \end{cases}$$

Since each alternative is non-inferior, so there exists no preference relation on the all the alternatives. Then, we aggregate the above optimization models with equal weights into the following optimization model of single objective,

$$(OM2)\left\{\min \xi_j = \sum_{j=1}^{m}\sum_{k=1}^{n}\left(\xi_{jk}^{-} - \xi_{jk}^{+}\right)\vartheta_k\right.$$

To finding solution of OM2, we obtain optimal solution $\vartheta = (\vartheta_1, \vartheta_2, ..., \vartheta_m)$, which utilized as weights information's of provided alternatives. Then, we obtain $\xi_j^{+}\left(l = 1,2,...,m\right)$ and $\xi_j^{-}\left(j = 1,2,...,m\right)$ as utilizing above formula, respectively.

Step: 7

Relative degree calculated for each alternative utilizing the following equation from PIS and NIS,

$$\xi_k = \frac{\xi_j^{+}}{\xi_j^{-} + \xi_j^{+}} \quad \left(j = 1,2,...,m\right).$$

Step: 8

Ranking all the alternatives A_j ($j=1,2,3,....,m$) and select finest one(s) in accordance with ξ_j ($j=1,2,3,....,m$). If any alternative has the highest ξ_j value, then it is finest alternative according to the criteria.

A Descriptive Example

Suppose a multi-national corporation in Pakistan is scheduling its economic stratagem for the imminent year, according to a group stratagem objective. For this, there are four alternatives attained after their initial scrutiny and are defined as ξ_1 to participate in the "Asian markets"; ξ_2 to participate in the "Western markets"; ξ_3 to participate in the "China markets"; and ξ_4 to participate in the" Local markets". This estimation ensues from the four characteristics, namely as g_1 "the evolution scrutiny"; g_2 "the danger scrutiny"; "the social-political influence scrutiny" and g_4 "the environmental influence scrutiny" whose weights information is $\omega = (0.2, 0.3, 0.1, 0.4)^T$.

	g_1	g_2	g_3	g_4
ξ_1	$\langle 0.2, 0.1, 0.6 \rangle$	$\langle 0.5, 0.3, 0.1 \rangle$	$\langle 0.5, 0.1, 0.3 \rangle$	$\langle 0.4, 0.3, 0.2 \rangle$
ξ_2	$\langle 0.1, 0.4, 0.4 \rangle$	$\langle 0.6, 0.3, 0.1 \rangle$	$\langle 0.5, 0.2, 0.2 \rangle$	$\langle 0.2, 0.1, 0.7 \rangle$
ξ_3	$\langle 0.3, 0.2, 0.2 \rangle$	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.4, 0.1, 0.3 \rangle$	$\langle 0.3, 0.3, 0.4 \rangle$
ξ_4	$\langle 0.3, 0.1, 0.6 \rangle$	$\langle 0.1, 0.2, 0.6 \rangle$	$\langle 0.1, 0.3, 0.5 \rangle$	$\langle 0.2, 0.3, 0.2 \rangle$

Step: 1

Weight the attribute value of each alternative by utilizing the operations of PFSs as,

	g_1	g_2	g_3	g_4
ξ_1	$\langle 0.043, 0.630, 0.902 \rangle$	$\langle 0.187, 0.696, 0.501 \rangle$	$\langle 0.066, 0.794, 0.886 \rangle$	$\langle 0.184, 0.617, 0.525 \rangle$
ξ_2	$\langle 0.020, 0.832, 0.832 \rangle$	$\langle 0.240, 0.696, 0.501 \rangle$	$\langle 0.066, 0.851, 0.851 \rangle$	$\langle 0.085, 0.398, 0.867 \rangle$
ξ_3	$\langle 0.068, 0.724, 0.724 \rangle$	$\langle 0.240, 0.510, 0.617 \rangle$	$\langle 0.049, 0.794, 0.886 \rangle$	$\langle 0.132, 0.617, 0.693 \rangle$
ξ_4	$\langle 0.068, 0.630, 0.902 \rangle$	$\langle 0.031, 0.617, 0.857 \rangle$	$\langle 0.010, 0.886, 0.933 \rangle$	$\langle 0.085, 0.617, 0.525 \rangle$

Step: 2

Calculate the score function by utilizing the definition 6 as,

$sc(\xi_1 g_1) = -0.85$	$sc(\xi_1 g_2) = -0.31$	$sc(\xi_1 g_3) = -0.81$	$sc(\xi_1 g_4) = -0.34$
$sc(\xi_2 g_1) = -0.81$	$sc(\xi_2 g_2) = -0.26$	$sc(\xi_2 g_3) = -0.78$	$sc(\xi_2 g_4) = -0.78$
$sc(\xi_3 g_1) = -0.65$	$sc(\xi_3 g_2) = -0.37$	$sc(\xi_3 g_3) = -0.83$	$sc(\xi_3 g_4) = -0.56$
$sc(\xi_4 g_1) = -0.83$	$sc(\xi_4 g_2) = -0.82$	$sc(\xi_4 g_3) = -0.92$	$sc(\xi_4 g_4) = -0.43$

Step: 3

Calculate PFPIS by utilizing score values,

$$P^+ = \begin{Bmatrix} \langle 0.068, 0.724, 0.724 \rangle, \langle 0.240, 0.696, 0.501 \rangle, \\ \langle 0.066, 0.851, 0.851 \rangle, \langle 0.184, 0.617, 0.525 \rangle \end{Bmatrix}$$

and similarly calculate PFNIS by utilizing score values

$$P^- = \begin{Bmatrix} \langle 0.043, 0.630, 0.902 \rangle, \langle 0.031, 0.617, 0.857 \rangle, \\ \langle 0.010, 0.886, 0.933 \rangle, \langle 0.085, 0.398, 0.867 \rangle \end{Bmatrix}$$

Step: 4

Utilizing the normalized Hamming distance to calculate Picture fuzzy positive-ideal separation matrix D^+ as,

$$\begin{vmatrix} 0.074 & 0.013 & 0.023 & 0.000 \\ 0.065 & 0.000 & 0.000 & 0.165 \\ 0.000 & 0.077 & 0.027 & 0.054 \\ 0.068 & 0.161 & 0.043 & 0.024 \end{vmatrix}$$

and to calculate Picture fuzzy negative-ideal separation matrix D as,

$$\begin{vmatrix} 0.000 & 0.148 & 0.048 & 0.165 \\ 0.073 & 0.161 & 0.043 & 0.000 \\ 0.074 & 0.141 & 0.044 & 0.110 \\ 0.006 & 0.000 & 0.000 & 0.140 \end{vmatrix}$$

Step: 5

Calculate the grey coefficient of each alternative from PIS as,

$$\xi_{jk}^{+} = \begin{vmatrix} 0.527 & 0.863 & 0.782 & 1.000 \\ 0.559 & 1.000 & 1.000 & 0.333 \\ 1.000 & 0.517 & 0.753 & 0.604 \\ 0.548 & 0.338 & 0.657 & 0.774 \end{vmatrix}$$

Calculate the grey coefficient of each alternative from NIS as,

$$\xi_{jk}^{-} = \begin{vmatrix} 1.000 & 0.358 & 0.632 & 0.333 \\ 0.530 & 0.338 & 0.657 & 1.000 \\ 0.527 & 0.369 & 0.652 & 0.428 \\ 0.932 & 1.000 & 1.000 & 0.370 \end{vmatrix}$$

Step: 6

Utilize the model (M2) to establish the following

$$\min \xi(w) = -0.849w_1 - 0.367w_2 - 0.898w_3 + 0.985w_4$$

Solve this model, we obtained the weight vectors of the attributes,

$$w = (0.263, 0.292, 0.095, 0.350)$$

After that, we can obtain the degree of grey coefficient for each alternative from PFPIS and PFNIS respectively, as,

	ξ_1^+	ξ_2^+	ξ_3^+	ξ_4^+
PFPIS	0.813	0.650	0.695	0.574
	ξ_1^-	ξ_2^-	ξ_3^-	ξ_4^-
PFNIS	0.543	0.649	0.455	0.761

Step: 7

Relative degree of each alternative calculated using the following equation from PIS and NIS,

$\xi_j = \frac{\xi_j^+}{\xi_j^- + \xi_j^+}$ $(j = 1,2,....,m)$ as $\qquad \xi_j = \frac{\xi_j^+}{\xi_j^- + \xi_j^+}$

$\xi_1 = 0.599$, $\xi_2 = 0.50$, $\xi_3 = 0.604$, $\xi_4 = 0.427$

Step: 8

According to relational degree, we are ranking the alternatives as

Score of the alternatives			
$\xi_1 = 0.599$	$\xi_2 = 0.50$	$\xi_3 = 0.604$	$\xi_4 = 0.427$
Ranking of the alternative according to score values			
$\xi_3 = 0.604 > \xi_1 = 0.599 > \xi_2 = 0.50 > \xi_4 = 0.427$			

Hence, the best alternative is ξ_3 = "China markets".

Comparison Analysis with Intuitionistic Fuzzy Grey Approach

Intuitionistic fuzzy numbers can describe the ambiguous things from degrees of positive and negative membership. They deliver an active implementation to signifies the indeterminacy of DM. On the one hand, as declared formerly, in IFN that mediating the things from good and bad features of these two varieties of fuzzy numbers can throw away the thoughts of DM exactly. However, dissimilar the PFNs, the IFNs are not serviceable in some conditions. The IFNs must gratify that the sum of the membership and non-membership degree belongs to [0,1]. Thus, in our case analysis, there exists some numbers which cannot handle by IFNs. For example, in the situation that human being needs opinions involving that type of answer: "yes", "abstain", "No" and "Refusal". Picture fuzzy set is useful in that situation. In summary, the PFNs have stronger capability to procedure information in decision theory, as compared to IFNs [18-33].

Conclusion

The grey relational procedure plays a dynamic role throughout the decision-making procedure and hence in this track, the comparative importance of the criteria remains un-reformed in modified problem. Almost all the researchers have operated the IFS by considering the positive and negative membership degrees only. But, it has been detected that in some circumstances, like in situation of voting, human thoughts including more degrees as, yes, neutral, no, refusal, and in which situation, IFS cannot be exactly characterized. For this circumstance, Picture fuzzy set, which is an upgrading of IFS, has utilized in this paper corresponding picture fuzzy aggregated operator are introduced. Several required characteristics of these operators have explored comprehensibly. Finally, a decision problem has established which based on these defined operators, for ranking the dissimilar alternatives by utilizing picture fuzzy environment. The suggested technique has demonstrated with a numerical example for viewing their effectiveness as well as reliability. Thus, the suggested operations give a new easier track to grip the inexact data throughout the decision problem procedure. In future, we utilize different decision making approaches like TOPSIS, TODAM, VIKOR, ELECTRIC-1, ELECTRICT-2 and many more to deal with uncertainty which is in the form of picture fuzzy sets in real life decision making problems.

References

1. Zadeh LA (1965) Fuzzy sets. Inf Control 8: 338-56.

2. Atanassov KT (1986) Intuitionistic fuzzy sets. Fuzzy Sets Syst 20: 87-96.

3. Xu Z (2007) Intuitionistic Fuzzy Aggregation Operators. IEEE Trans Fuzzy Syst 15: 1179-87.

4. Xu Z, Yager RR (2006) Some geometric aggregation operators based on intuitionistic fuzzy sets. Int J Gen Syst 35.

5. Yager RR (2013) Pythagorean fuzzy subsets. In: Proc Joint IFSA World Congress and NAFIPS Annual Meeting, Edmonton, Canada 57-61.

6. Cuong BC, Kreinovitch V (2013) Picture fuzzy sets - A new concept for computational intelligence problems. Departmental Technical Reports (CS), Paper 809.

7. Garg H (2017) Some Picture Fuzzy Aggregation Operators and Their Applications to Multicriteria Decision-Making, Arab J Sci Eng 42: 5275-90.

8. Singh P (2015) Correlation coefficients for picture fuzzy sets. J Intell Fuzzy Syst 28: 591-604.

9. Wei G (2016) Picture fuzzy cross-entropy for multiple attribute decision making problems. J Bus Econ Manage 17: 491-502.

10. Ashraf S, Mahmood T, Abdullah S, Khan Q (2018) Different Approaches to Multi-Criteria Group Decision Making Problems for Picture Fuzzy Environment, Bulletin of the Brazilian Mathematical Society, New Series.

11. Zeng S, Ashraf S, Arif, M, Abdullah S (2019) Application of Exponential Jensen Picture Fuzzy Divergence Measure in Multi-Criteria Group Decision Making. Math 7: 191.

12. Khan MJ, Kumam P, Ashraf S, Kumam W (2019) Generalized Picture Fuzzy Soft Sets and Their Application in Decision Support Systems. Symmetry 11: 415.

13. Ashraf S, Abdullah S, Qadir A (b2018) Novel concept of cubic picture fuzzy sets. J New Theory 24: 59-72.

14. Ashraf S, Mahmood T, Abdullah S, Khan Q (2018) Picture Fuzzy Linguistic Sets and Their Applications for Multi-Attribute Group Decision Making Problems. The Nucleus 55: 66-73.

15. Wei GW (2010) GRA method for multiple attribute decision making with incomplete weight information in intuitionistic fuzzy setting. Knowledge-Based Syst 23: 243-7.

16. Wei GW (2011) Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making, Expert Syst Appl 38: 11671-7.

17. Kim SH, Ahn BS (1999) Interactive group decision making procedure under incomplete information. Eur J Oper Res 116: 498-507.

18. Ashraf S, Abdullah S, Mahmood T (2018) GRA method based on spherical linguistic fuzzy Choquet integral environment and its application in multi-attribute decision-making problems. Math Sci 12: 263-75.

19. Atanassov KT (1994) Operations over interval-valued fuzzy set. Fuzzy Sets Syst 64: 159-74.

20. Atanassov KT (2015) Intuitionistic fuzzy logics as tools for evaluation of data mining processes. Knowl-Based Syst 80: 122-30.

21. Atanassov KT (1994) New operations defined over the intuitionistic fuzzy sets, Fuzzy Sets Syst 61: 137-142.

22. Atanassov KT (1999) Intuitionistic fuzzy sets, springer, Heidelberg.

23. Bujnowski P, Szmidt E, Kacprzyk J (2014) Intuitionistic fuzzy decision tree: a new classifier. Intell Syst 779-90.

24. Cuong BC (2014) Picture fuzzy sets. J Comput Sci Cybern 30: 409-20.

25. Cuong BC (2013) Picture fuzzy sets first results. part 1, Seminar Neuro-Fuzzy Systems with Applications, Preprint 03/2013, Institute of Mathematics, Hanoi.

26. Deng JL (1989) Introduction to grey system theory. Grey Syst 1: 1-24.

27. Deng JL (2001) Grey system theory. Press of Huazhong University of Science and Technology, Wuhan.

28. Khan AA, Ashraf S, Abdullah S, Qiyas M, Luo J, et al. (2019) Pythagorean Fuzzy Dombi Aggregation Operators and Their Application in Decision Support System. Symmetry 11: 383.

29. Qiyas M, Abdullah S, Ashraf S (2019) Solution of multi-criteria group decision making problem based on picture linguistic information's. Int J Algebra Statis 8: 1-11.

30. Liu C, Tang G, Liu P (2017) An Approach to Multi-criteria Group Decision-Making with Unknown Weight Information Based on Pythagorean Fuzzy Uncertain Linguistic Aggregation Operators. Math Prob Eng 2017: 6414020.

31. Wei G (2017) Some Cosine Similarity Measures for Picture Fuzzy Sets and Their Applications to Strategic Decision Making. Informatica, 28: 547-64.

32. Zeng S (2017) Pythagorean Fuzzy Multi attribute Group Decision Making with Probabilistic Information and OWA Approach. Int J Intell Syst 1-15.

33. Zhang X, Jin F, Liu P (2013) A grey relational projection method for multi-attribute decision making based on intuitionistic trapezoidal fuzzy number. Appl Math Modell 37: 3467-77.