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Comparison of Balance Coefficient Methods in Efficient Fractional Factorial Design Using Generalized Minimum Aberration (GMA) and Minimum Moment Aberration (MMA)

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Abstract

Efficient orthogonal arrays with three factors having two, three and four levels were constructed with balance and orthogonal property for lowest common multiples of runs. The two forms of balance coefficient were used for classifying the designs into two; and minimum aberration criteria were used to determine designs with less aberration. The designs constructed using the maximum form of balance coefficient has the less aberration in both the Generalized Minimum Aberration and Minimum Moment Aberration criteria.

Keywords: Balance Coefficient; Generalized Minimum Aberration (GMA); Minimum Moment Aberration (MMA); Fractional Factorial; J_2 Optimality

Introduction

Orthogonal designs are commonly used in applied work because of their optimality properties, ease of analysis and interpretation. Traditionally, the construction of orthogonal Sn fractional factorial designs has been confined to the class of designs defined by appropriately chosen aliasing relations. A two-level 2^{m-q} design is defined to be a fractional factorial design with m factors, each at two levels, consisting of 2^{m-q} runs. Therefore, it is a 2^{-q} fraction of the 2^m full factorial design in which the fraction is determined by q generators, where a generator consists of letters which are the names of the factors denoted by A, B and so on. The number of letters in a word is its word length and the word formed by the q defining words is called the defining relation.

For a 2^{m-q} design, let A_K (d) be the number of words of length k in the defining contrast subgroup. The vector

is called the word length pattern of the design d (Fries and Hunter, 1980). The resolution of a 2^{m-q} design, R, is defined to be the smallest r such that $A_r(d) \geq 1$, that is, the length of the shortest word in the defining contrast subgroup for any two 2^{m-q} designs d_1 and d_2, let r be the smallest integer such that $A_r(d_1) \neq Ar(d_2)$. Then d1 is said to have less aberration than d_2 if $Ar(d_1) < Ar(d_2)$. If no design exists with less aberration than d1, then d1 has minimum aberration.

$$W(d) = (A_1(d), A_2(d), \ldots, A_m(d))$$

Consider a 2^{7-2} experiment, with three design options which provides the design generators for three designs along with their defining relations. In this example, d_3 has less aberration than d_1 or d_2 because the first unequal number in word length pattern is in the fourth position and d_3 has the smallest number in that position. Design d_3 is the minimum aberration 2^{7-2} design. Other 2^{m-q} minimum aberration designs and their design generators are presented in Montgomery (2001) [1]. Montgomery (2001) gives a slightly different formatted word length pattern from Wu and Zhang (1993), instead of using numbers of words of length k in the defining contrast subgroup, Montgomery (2005) directly shows the length of each word in the defining contrast group [1,2].

Minimum aberration mixed-level designs are also balanced, Cheng et al, (1999), Deng and Tang (1999), Mukerjee and Wu (2001), (Xu and Wu (2001) [3-5]. For unbalanced mixed-level fractional factorial designs, the degree of balance was evaluated using a balance coefficient (Guo (2003)). As an extension of two level fractional factorial designs, Franklin (1984) and Suen, Chen and

Wu (1997) discuss the construction of multi-level minimum aberration designs [7,8]. Xu and Wu (2001) proposed a generalized minimum aberration for mixed –level fractional factorial designs [9]. Wu and Zhang (1993) and Ankenman (1999) used minimum aberration designs in two and four - level designs. Murkerjee and Wu (2001) developed minimum aberration designs for mixed-level fractional factorial designs involving factors with two or three distinct levels. The objective of this paper is to compare designs generated by methods of balance (balance coefficient) in an efficient mixed level fractional factorial designs using Generalized Minimum Aberration Criteria and Minimum Moment Aberration at various runs sizes.

Balance Coefficient of Form I

In form I, the motivation behind the definition of the balance coefficient is a simple optimization problem. The balance coefficient of design matrices will be derived from the optimization problem stated below:

Max $G = \prod_{K=1}^{m} X_K$

Subject to $\sum_{k=1}^{m} X_K = C$

Where C is a constant

The balance coefficient for design matrix k, *F (k)*, is defined as the combination of the balanced coefficient of each column, F_j

$$F(k) = \sum_{j=1}^{m} w_j F_J = \sum_{j=1}^{m} \left(\prod_{i=1}^{l_j} l_{ij} \right) w_j ,$$

Where w_j are the weights for the corresponding column *j*. This balance coefficient depends on the runs. To avoid this situation, a standardized balance coefficient is defined by using a standardized number of levels. The balanced coefficient is standardized when the number of levels is standardized. The notations f_{ij} is used instead of l_{ij}.

Balance Coefficient of Form II

In form II, the definition of balance coefficient employs the concept of the distance function. Consider a distance function

$$H_J = \sum_{i=1}^{lj} (l_{ij} - T)^2 ,$$

where $T = n/l_j$, is a fixed value?

The balance coefficient under this definition becomes

$$H_j = \sum_{i=1}^{l_j} (l_{ij} - \frac{n}{l_j})^2$$

and

$$H = \sum_{j=1}^{m} H_j = \sum_{j=1}^{m} w_j \sum_{i=1}^{l_j} (l_{ij} - \frac{n}{l_j})^2$$

If f_{ij} are used instead of l_{ij}, then standardized H_j and H can be given by

$$\hat{H}_j = \sum_{i=1}^{l_j} \text{ü} f_{ij} - \frac{1}{l_j} \; ^2 \quad \text{and}$$

$$\hat{H} = \sum_{j=1}^{m} w_j \hat{H}_j = \sum_{j=1}^{m} w_j \sum_{i=1}^{l_j} (f_{ij} - \frac{1}{l_j})^2$$

Minimum Aberrations

Minimum aberration has been widely recognized as a useful criterion for selecting regular fractional factorials. Recent work on minimum aberration designs includes Chen and Wu (2001), Tang and Wu (1996), Chen and Hedayat (1996), and Cheng *et al.* (1999) [10-12].

Generalized Minimum Aberration Criterion

Xu and Wu (2001) proposed a generalized minimum aberration *(GMA)* criterion for multi-level and mixed-level designs. For a design *d*, the ANOVA model has the following form

$$Y = X_0\alpha_0 + X_1\alpha_1 + \ldots + X_m\alpha_m + \varepsilon,$$

where *Y* is the response, α_k is the vector of all k-factor interactions and $X_k = \left[x_{ij}^{(k)} \right]$ is the matrix of contrast coefficients for α_k. Let

$$A_k(d) = n^{-2}\sum_j \left| \sum_{i=1}^{n} x_{ij}^{(k)} \right|^2 .$$

The $A_k(d)$ are invariant with respect to the choice of orthogonal contrasts. The vector $\left(A_1(d), A_2(d), \ldots A_m(d) \right)$ is called the generalized word length pattern. Then the generalized minimum aberration criterion is to sequentially minimize $A_k(d)$ for $k=1, \ldots, m$

Minimum Moment Aberration Criterion

The Minimum Generalized Aberration *(MGA)*, Minimum G_2 Aberration *(MG2A)*, and Generalized Minimum Aberration *(GMA)* criteria all require contrast coefficients of factors. Xu (2003) developed a Minimum Moment Aberration criterion (MMA), which does not need contrast coefficients. For a design matrix *d*, with d_{ij} as the elements of i^{th} row and j^{th} column. The coincidence between two elements d_{ij} and d_{lj} is defined by $\delta\left(d_{ij}, d_{lj}\right)$, where $\delta\left(d_{ij}, d_{lj}\right) = 1$ if $d_{ij} = d_{lj}$ and 0 otherwise. The value of $\sum_{j=1}^{m}\delta\left(d_{ij}, d_{lj}\right)$ measures the coincidence between i^{th} and j^{th} rows of *d*. The k^{th} power moment is defined by Xu (2003) as

$$K_k\left(d\right) = \left[n(n-1)/2\right]^{-1} \sum_{1 \le i \le l \le n} \left[\sum_{j=1}^{m} \left(d_{ij}, d_{lj}\right) \right]^k .$$

For two designs d_1 and d_2, d_1 is said to have less moment aberration than d_2 if there exists an r such that $K_r(d_1) < K_r(d_2)$ and $K_t(d_1) = K_t(d_2)$ for all $t=1, \ldots, r-1$. Therefore, d_1 is said to have minimum moment aberration if there is no other design with less moment aberration than d_1 (Table 1, 2 and 3).

		Generalized Minimum Aberration Criteria		
Runs	Designs	Sum of Squares	$A_i(d_i)$	Decision
6	Min.	16.99	0.472	d_2
	Max.	2.99	0.083	
7	Min.	29.89	0.61	d_2
	Max.	3.92	0.08	
8	Min.	46.04	0.72	d_2
	Max.	0.67	0.01	
9	Min.	67.21	0.82	d_2
	Max.	3.24	0.04	
10	Min.	91.62	0.92	d_2
	Max.	1.67	0.02	
11	Min.	103.86	0.86	d_2
	Max.	5.91	0.05	
12	Min.	103.95	0.72	d_2
	Max.	0	0	
13	Min.	91.87	0.54	d^2
	Max.	3.92	0.02	

		Generalized Minimum Aberration Criteria		
Runs	Designs	Sum of Squares	$A_1(d_1)$	Decision
14	Min.	83.63	0.43	d_2
	Max.	5.67	0.03	
15	Min.	79.22	0.35	d_2
	Max.	11.25	0.05	
16	Min.	70.64	0.28	d_2
	Max.	34.66	0.14	
17	Min.	77.89	0.27	d_2
	Max.	5.91	0.02	
18	Min.	88.97	0.27	d_2
	Max.	4.99	0.02	

Table 1: Design Comparison using Generalized Minimum Aberration Criterion (GMAC)

The generalized minimum aberration criteria in the comparison of the designs, $OA(n, 2^1 3^1 4^1)$, using both form I (Max.) and II (Min.) methods of balance coefficient for $6 \le n \le 18$, it was shown that at $6 \le n \le 18$, $A_1(d_2) < A_1(d_1)$, i.e. design d_2 has less aberration than d_1. Therefore, d_2 is better than d1 by the GMAC.

		Minimum Moment Aberration Criteria	
Runs(N)	Designs	$(K_1(d),\ K_2(d),\ K_3(d),\ K_4(d))$	Decision
6	Min.	(1.267, 2.6, 5.667, 13)	d_2
	Max.	(0.8, 1.6, 3.6, 8.8)	
7	Min.	(1.429, 3.667, 9.476, 26.43)	d2
	Max.	(0.857, 1.714, 4.809, 10.286)	
8	Min.	(1.643, 4.143, 11.286, 32)	d2
	Max.	(0.821, 1.679, 3.964, 10.607)	
9	Min.	(1.75, 4.611, 12.778, 36.611)	d2
	Max.	(0.88, 1.94, 4.88, 13.28)	
10	Min.	(1.889, 4.467, 11.622, 31.933)	d2
	Max.	(0.88, 1.82, 3.67, 10.62)	
11	Min.	(1.727, 4.2, 10.545, 28.2)	d2
	Max.	(0.872, 2.036, 4.873, 12.509)	
12	Min.	(1.697, 4.181, 11.060, 30.879)	d2
	Max.	(0.909, 1.939, 4.727, 12.485)	
13	Min.	(1.513, 3.487, 8.897, 24.103)	d2
	Max.	(1.025, 2.077, 5.103, 13.462)	
14	Min.	(2.494, 2.978, 7.132, 17.703)	d2
	Max.	(1, 2.418, 10.099, 16.923)	
15	Min.	(1.352, 2.971, 7.038, 18.371)	d2
	Max.	(0.96, 2.37, 6.314, 17.457)	
16	Min.	(1.267, 2.75, 7.05, 19)	d2
	Max.	(1.1, 2.608, 6.55, 17.483)	
17	Min.	(1.235, 6.169, 9.279, 26.779)	d2
	Max.	(0.98, 4.93, 5.78, 15.93)	
18	Min.	(1.261, 3.366, 10.189, 26.634)	d2
	Max.	(0.987, 2.229, 5.693, 15.562)	

Table 2: Designs using Minimum Moment Aberration Criteria (MMA) in $(n, 2^1 3^1 4^1)$

The minimum aberration criteria for two selected designs using form I (Max.) and form II (Min.) method of balance coefficient, for $6 \le n \le 18$.

The observation shows that at $6 \le n \le 18$

$$K_1(d_2) < K_1(d_1)$$

This indicated that in all the runs mentioned, K_1 (d_2) has a lesser aberration than K_1 (d_2), that is, the design d_2 is a better fractional factorial of all possible designs in the runs considers.

	OA $(n, 2^1 3^1 4^1)$						
Forms	**Runs (N)**	**MMAC**	**GMAC**	**Forms**	**Runs (N)**	**MMAC**	**GMAC**
Max.	**6**	d_2	d_2	Max.	**13**	d^2	d^2
Max.	**7**	d^2	d_2	Max.	**14**	d_2	d^2
Max.	**8**	d_2	d_2	Max.	**15**	d_2	d^2
Max.	**9**	d_2	d_2	Max.	**16**	d_2	d_2
Max.	**10**	d_2	d_2	Max.	**17**	d_2	d_2
Max.	**11**	d_2	d_2	Max.	**18**	d_2	d_2
Max.	**12**	d_2	d_2				

Table 3: Summary of designs evaluated in Table 1 and 2

Conclusion

In this paper, constructed efficient fractional factorial designs with balance coefficient and J_2 optimality criteria were used to compare the two forms of balanced coefficient methods using the Generalized Minimum Aberration (GMA) and Minimum Moment Aberration (MMA) criteria. It was observed that designs constructed using the maximum form of balance coefficient has the less aberration in both the Generalized Minimum Aberration and Minimum Moment Aberration criteria.

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