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Effect of Rotation and Gravity on Generalized Thermoelastic Medium with Double Porosity under L-S Theory

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Abstract

In this paper, the discussion will be on the physical quantities of generalized thermoelastic medium with double porosity under Lord and Shulman theory. The effect of rotation and gravity has been established. The half-space is considered of an isotropic homogeneous thermoelastic material. The numerical results are discussed graphically with comparisons in the presence and absence of the rotation field by taking the solution method in the form of the exponential function.

Keywords: L-S Theory; Thermoelastic Medium; Rotation; Gravity; Double Porosity; Normal Mode; Plane Waves

Introduction

The generalized theory of thermoelasticity is proposed by Lord-Shulman (1967) and is known as (L-S) theory which involves one relaxation time for a thermoelastic process [1]. The basis of the model proposed by Lord and Shulman was to modify Fourier's law of the heat conduction equation by introducing a new physical concept which called a relaxation time needed for acceleration of the heat flow. The heat equation of this theory of the wave type, it automatically ensures finite speeds of propagation of heat and elastic waves. The remaining governing equations for this theory, namely, the equations of motions and constitutive relations, remain the same as,

$$\sigma_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} - \beta\delta_{ij}T.$$
$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} - \beta T_{,i} + F_i = \rho\ddot{u}_i.$$

Thus, the heat conduction equation, for isotropic homogeneous body, based on (L-S) theory is given by:

$$KT_{,ii} = (1+\tau_0\frac{\partial}{\partial t})[\rho C_E\frac{\partial T}{\partial t} + \beta T_0\frac{\partial e}{\partial t}].$$

Where τ_0 is the relaxation time, the time lag needed to establish steady state heat conduction in a volume element when a temperature gradient is suddenly imposed on the element, satisfying the condition $\tau_0 > 0$.

The equation of motion and heat equation with double porosity functions:

$$\sigma_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\Phi + d\delta_{ij}\Psi - \beta\delta_{ij}T.$$
$$\mu u_{i,jj} + (\lambda+\mu)u_{j,ij} + b\Phi_{,i} + d\Psi_{,i} - \beta T_{,i} + F_i = \rho\ddot{u}_i.$$
$$K^*\nabla^2 T = (1+\tau_0\frac{\partial}{\partial t})(\rho C^*\dot{T} + \beta T_0\dot{e} + \gamma_1 T_0\dot{\Phi} + \gamma_2 T_0\dot{\Psi}).$$

Propagation of the photothermal waves in a semiconducting medium under L-S theory by Othman *et al.* [2]. Othman *et al.* discussed the effect of the gravity on the photothermal waves in a semiconducting medium with an internal heat source and one relaxation time [3]. In the past some researchers have investigated different problems of rotating media. The propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied [4]. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. An investigation of the distribution of deformation, stresses and magnetic field in a uniformly rotating, homogeneous, isotropic, thermally and electrically conducting elastic half-space was presented [5]. The effect of rotation on elastic waves has been studied [6,7]. The effect of rotation in a magneto-thermoelastic medium was discussed [8].

The origin of the linear theory of elastic materials with double porosity goes back to papers of Barenblatt *et al.* [9,10]. The theory of flow and deformation in double porous media was used by Khalili and Valliappan [11]. Masters Pao, and Lewis studied coupling temperature to a double porosity model of deformable porous media [12]. Khalili and Selvadurai studied the fully coupled constitutive model for thermo-hydro-mechanical analysis in elastic media with double porosity [13]. Zhao and Chen introduced the fully coupled dual-porosity model for anisotropic formations [14]. The dynamical problems of the theory of elasticity for solids with double porosity were studied by Svanadze [15]. Ainouz investigated the homogenized double porosity models for poro-elastic media with interfacial flow Barrier [16]. Plane waves and boundary value problems in the theory of elasticity for solids with double porosity were studied by Svanadze [17]. Straughan studied the stability and uniqueness in double porosity elasticity. Mahmood *et al.* investigated the combined higher order finite volume and finite element scheme for double porosity and non-linear adsorption of transport problem in porous media [18,19]. Some researches in the past have investigated different problems of gravity field. Othman *et al.* applied the normal mode analysis on two-dimensional electro-magneto-thermoelastic plane wave problem of a medium of perfect conductivity [20-22]. In the present paper, we have discussed a homogeneous thermoelastic half-space with double porosity structure rotating uniformly with angular velocity and the effect of gravity, the equations of generalized thermoelastic material with double porosity structure with one relaxation time has been developed. Analytic solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed. The effect of porosity and rotation is shown numerically graphically.

Formulation of the Problem and Basic Equations

We consider a homogeneous thermoelastic half-space with double porosity structure rotating uniformly with angular velocity $\Omega = \Omega\, \boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation in the rotating frame has two additional terms [Schoenberg and Censor (1973)]: Centripetal acceleration $\Omega\times(\Omega\times u)$ due to time varying motion only and Coriolis acceleration $2\Omega\times\dot{u}$ where $u=(u,0,w)$ is the dynamic displacement vector and angular velocity $\Omega=(0,\Omega,0)$. These terms, do not appear in non-rotating media.

In case of isotropic solids, the constitutive equations for double porosity

$$\sigma_j = \alpha\,\Phi_{,j} + b_1\,\Psi_{,j} \tag{1}$$

$$\tau_j = b_1\,\Phi_{,j} + \gamma\,\Psi_{,j} \tag{2}$$

$$\xi = -\,be_{jj} - \alpha_1\Phi - \alpha_3\Psi + \gamma_1 T \tag{3}$$

$$\zeta = -\,de_{jj} - \alpha_3\Phi - \alpha_2\Psi + \gamma_2 T \tag{4}$$

Where ξ and ζ satisfy the equations

$$\sigma_{j,j} + \xi + \rho_0 g' = K_1\ddot{\Phi} \tag{5}$$

$$\tau_{j,j} + \zeta + \rho_0 L' = K_2\ddot{\Psi} \tag{6}$$

From (3), (4) in (5), (6) with the absence of body force to macro pores and fissures

$$\sigma_{j,j} - be_{jj} - \alpha_1\Phi - \alpha_3\Psi + \gamma_1 T = K_1\ddot{\Phi} \tag{7}$$

$$\tau_{j,j} - de_{jj} - \alpha_3\Phi - \alpha_2\Psi + \gamma_2 T = K_2\ddot{\Psi} \tag{8}$$

Stress equation

$$\sigma_{ij} = \lambda e_{rr}\delta_{ij} + 2\mu e_{ij} + b\delta_{ij}\Phi + d\delta_{ij}\Psi - \beta\delta_{ij}T \quad (9)$$

Where, $e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}),\ \omega_{ij} = \frac{1}{2}(u_{j,i} - u_{i,j})$.
From (1) in (7) and (2) in (8)

$$\alpha\,\Phi_{,jj} + b_1\Psi_{,jj} - be_{jj} - \alpha_1\Phi - \alpha_3\Psi + \gamma_1 T = K_1\ddot{\Phi} \quad (10)$$

$$b_1\Phi_{,jj} + \gamma\,\Psi_{,jj} - de_{jj} - \alpha_3\Phi - \alpha_2\Psi + \gamma_2 T = K_2\ddot{\Psi} \quad (11)$$

Equations of motion with the components of rotation and gravity

$$\mu\nabla^2 u + (\lambda+\mu)\frac{\partial e}{\partial x} + b\frac{\partial \Phi}{\partial x} + d\frac{\partial \Psi}{\partial x} - \beta\frac{\partial T}{\partial x} + \rho g\frac{\partial w}{\partial x} = \rho[\ddot{u} - \Omega^2 u + 2\Omega\dot{w}] \quad (12)$$

$$\mu\nabla^2 w + (\lambda+\mu)\frac{\partial e}{\partial z} + b\frac{\partial \Phi}{\partial z} + d\frac{\partial \Psi}{\partial z} - \beta\frac{\partial T}{\partial z} - \rho g\frac{\partial u}{\partial x} = \rho[\ddot{w} - \Omega^2 w - 2\Omega\dot{u}] \quad (13)$$

Equations of heat

$$K^*\nabla^2 T = (1 + \tau_0\frac{\partial}{\partial t})(\rho C^*\dot{T} + \beta T_0\dot{e} + \gamma_1 T_0\dot{\Phi} + \gamma_2 T_0\dot{\Psi}) \quad (14)$$

For the purpose of numerical evaluation, we introduce dimensionless variables

$$(x', z') = \frac{\omega_1}{c_1}(x,z),\ (u', w') = \frac{\omega_1}{c_1}(u,w),\ \{\sigma_1', \tau_1'\} = \frac{c_1}{\alpha\omega_1}\{\sigma_1, \tau_1\},\ (t', \tau_0') = \omega_1(t, \tau_0),$$

$$[\Phi', \Psi'] = \frac{K_1\omega_1^2}{\alpha_1}[\Phi, \Psi],\ c_1^2 = \frac{\lambda + 2\mu}{\rho},\ \omega^* = \frac{\rho c_e c_1^2}{K},\ \nabla^2 = \frac{\omega_1^2}{c_1^2}\nabla'^2,\ \gamma = (3\lambda + 2\mu)\alpha_t,$$

$$t_{ij}' = (\frac{1}{\beta t_0})t_{ij},\ T' = \frac{T}{T_0},\ g' = \frac{g}{c_1\omega_1},\quad \Omega' = \frac{\Omega}{\omega_1}.$$

Using the above dimensionless quantities, Eqs. (10) - (14) become:

$$\frac{(\lambda+\mu)}{\rho c_1^2}\frac{\partial e}{\partial x} + (\frac{\mu}{\rho c_1^2})\nabla^2 u + a_1\frac{\partial \Phi}{\partial x} + a_2\frac{\partial \Psi}{\partial x} - a_3\frac{\partial T}{\partial x} + g\frac{\partial w}{\partial x} = [\ddot{u} - \Omega^2 u + 2\Omega\dot{w}] \quad (15)$$

$$\frac{(\lambda+\mu)}{\rho c_1^2}\frac{\partial e}{\partial z} + (\frac{\mu}{\rho c_1^2})\nabla^2 w + a_1\frac{\partial \Phi}{\partial z} + a_2\frac{\partial \Psi}{\partial z} - a_3\frac{\partial T}{\partial z} - g\frac{\partial u}{\partial x} = [\ddot{w} - \Omega^2 w - 2\Omega\dot{u}] \quad (16)$$

$$a_4\nabla^2 T = (1 + \tau_0\frac{\partial}{\partial t})(\dot{T} + a_5\dot{e} + a_6\dot{\Phi} + a_7\dot{\Psi}) \quad (17)$$

$$a_8\nabla^2\Phi + a_9\nabla^2\Psi - a_{10}e - a_{11}\Phi - a_{12}\Psi + a_{13}T = \ddot{\Phi} \quad (18)$$

$$a_{14}\nabla^2\Phi + a_{15}\nabla^2\Psi - a_{16}e - a_{17}\Phi - a_{18}\Psi + a_{19}T = \ddot{\Psi} \tag{19}$$

Where,

$$a_1 = \frac{b\alpha_1}{\rho c_1^2 K_1 w_1^2},\ a_2 = \frac{d\alpha_1}{\rho c_1^2 K_1 w_1^2},\ a_3 = \frac{\beta T_0}{\rho c_1^2},\ a_4 = \frac{K w_1}{\rho c c_1^2},\ a_5 = \frac{\beta}{\rho c},\ a_6 = \frac{\gamma_1\alpha_1}{\rho c K_1 w_1^2},$$
$$a_7 = \frac{\gamma_2\alpha_1}{\rho c K_1 w_1^2},\ a_8 = \frac{\alpha}{c_1^2 K_1},\ a_9 = \frac{b_1}{c_1^2 K_1},\ a_{10} = \frac{b}{\alpha_1},\ a_{11} = \frac{\alpha_1}{K_1 w_1^2},\ a_{12} = \frac{\alpha_3}{K_1 w_1^2},\ a_{13} = \frac{\gamma_1 T_0}{\alpha_1},$$
$$a_{14} = \frac{b_1}{c_1^2 K_2},\ a_{15} = \frac{\gamma}{c_1^2 K_2},\ a_{16} = \frac{dK_1}{\alpha_1 K_2},\ a_{17} = \frac{\alpha_3}{w_1^2 K_2},\ a_{18} = \frac{\alpha_2}{w_1^2 K_2},\ a_{19} = \frac{\gamma_2 T_0 K_1}{\alpha_1 K_2}.$$

We define displacement potentials ϕ_1 and ψ_1 which relate to displacement components u_1 and u_3 as,

$$u = \phi_{1,x} - \o_{1,z},\ \mathrm{w} = \phi_{1,z} + \o_{1,x} \tag{20}$$

Using Eq. (20) in Eqs. (15) - (19), we obtain:

$$\nabla^2\phi_1 + a_1\Phi + a_2\Psi - a_3T + g\psi_1 = [\ddot{\phi}_1 - \Omega^2\phi_1 + 2\Omega\psi_1] \tag{19}$$

$$(\frac{\mu}{\rho c_1^2})\nabla^2\psi_1 + g\frac{\partial\phi_1}{\partial x} = [-\ddot{\psi}_1 + \Omega^2\psi_1 + 2\Omega\dot{\phi}_1] \tag{20}$$

$$a_4\nabla^2 T = (1+\tau_0\frac{\partial}{\partial t})(\dot{T} + a_5\nabla^2\dot{\phi}_1 + a_6\dot{\Phi} + a_7\dot{\Psi}) \tag{21}$$

$$a_8\nabla^2\Phi + a_9\nabla^2\Psi - a_{10}\nabla^2\phi_1 - a_{11}\Phi - a_{12}\Psi + a_{13}T = \ddot{\Phi} \tag{22}$$

$$a_{14}\nabla^2\Phi + a_{15}\nabla^2\Psi - a_{16}\nabla^2\phi_1 - a_{17}\Phi - a_{18}\Psi + a_{19}T = \ddot{\Psi} \tag{23}$$

Dimensionless variables of the stress components take the form,

$$\sigma_{xx} = (\frac{\lambda}{\beta T_0})\nabla^2\phi_1 + (\frac{2\mu}{\beta T_0})\frac{\partial u}{\partial x} - T + (\frac{b\alpha_1}{K_1 w_1^2\beta T_0})\Phi + (\frac{d\alpha_1}{K_1 w_1^2\beta T_0})\Psi \tag{24}$$

$$\sigma_{zz} = (\frac{\lambda}{\beta T_0})\nabla^2\phi_1 + (\frac{2\mu}{\beta T_0})\frac{\partial w}{\partial z} - T + (\frac{b\alpha_1}{K_1 w_1^2\beta T_0})\Phi + (\frac{d\alpha_1}{K_1 w_1^2\beta T_0})\Psi \tag{25}$$

$$\sigma_{xz} = (\frac{\mu}{\beta T_0})[\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}] \tag{26}$$

The solution of the considered physical variable can be taken in the form

$$[T, \phi_1, \psi_1, \Phi, \Psi, \sigma_{ij}](x,z,t) = [T^*, \phi_{11}^*, \psi_1^*, \Phi^*, \Psi^*, \sigma_{ij}^*](z)\exp[i(\omega t + ax)] \tag{27}$$

Where, ω is the complex time constant (frequency), *i* is the imaginary unit, and α is the wave number in x-direction.

Using (27) in Eqs. (19) – (23), we obtain

$$(\mathrm{D}^2 + B_1)\phi_1^* + a_1\Phi^* + a_2\Psi^* - a_3T^* + B_2\psi_1^* = 0 \tag{28}$$

$$B_3\psi_1^* + B_3\phi_1^* = 0 \tag{29}$$

$$(\mathrm{a}_4\,\mathrm{D}^2 - B_5)T^* - B_6(\mathrm{D}^2 - a^2)\phi_1^* - B_7\Phi^* - B_8\Psi^* = 0 \tag{30}$$

$$(\mathrm{a}_8\,\mathrm{D}^2 - B_9)\Phi^* + (a_9\mathrm{D}^2 - B_{10})\Psi^* - (a_{10}\mathrm{D}^2 - B_{11})\phi_1^* + a_{13}T^* = 0 \tag{31}$$

$$(\mathrm{a}_{14}\,\mathrm{D}^2 - B_{12})\Phi^* - (a_{16}\mathrm{D}^2 - B_{14})\phi_1^* + (a_{15}\mathrm{D}^2 - B_{13})\Psi^* + a_{19}T^* = 0 \tag{32}$$

Where, $B_1 = (-a^2 + \omega^2 + \Omega^2),\ B_2 = g + 2i\Omega\omega,\ B_3 = \dfrac{\mu}{\rho c_1^2} - \omega^2 - \Omega^2,\ B_4 = iag - 2i\Omega\omega,$, $B_5 = \mathrm{a}^2\,\mathrm{a}_4 + \mathrm{i}\omega(1 + \mathrm{i}\omega\tau_0),$
$B_6 = \mathrm{i}\omega\,\mathrm{a}_5(1 + \mathrm{i}\omega\tau_0),\ B_7 = \mathrm{i}\omega\,\mathrm{a}_6(1 + \mathrm{i}\omega\tau_0),\ B_8 = \mathrm{i}\omega\,\mathrm{a}_7(1 + \mathrm{i}\omega\tau_0),\ B_9 = (a_8a^2 + a_{11} - \omega^2),$
$B_{10} = a_9a^2 + a_{12},\ B_{11} = a_{10}a^2,\ B_{12} = (a_{14}a^2 + a_{17}),\ B_{13} = (a_{15}a^2 + a_{18} - \omega^2),\ B_{14} = a_{16}a^2.$

By solving Eqs. (28) - (31) in the matrix, we get

$$[D^8 - AD^6 + B\,D^4 - C\,D^2 + E]\{\phi_1^*(z), \Phi^*(z), \psi^*(z), \psi_1^*(z), T^*\} = 0 \tag{33}$$

Where,

$$\begin{aligned}
A = {} & (a_1a_{10}B_3a_{15}a_4 - a_1a_{16}B_3a_9a_4 - a_8B_3a_{15}B_5 - a_8B_3a_{13}B_4 + a_8B_1a_{15}B_3a_4 - a_8B_4a_{15}B_2a_4 \\
& - a_8B_6a_3B_3a_{15} + a_8a_{16}a_2B_3a_4 - B_9B_{15}B_3a_4 + a_{14}B_3B_5a_9 + a_{14}B_3B_{10}a_4 - a_{14}B_3B_1a_4a_9 \\
& + a_{14}B_4B_2a_4a_9 + a_{14}B_3B_6a_3a_9 - a_{14}B_3a_2a_4a_{10} + a_4B_{12}B_3a_9) / (a_{14}B_5B_3a_9 - a_4B_3a_{15}a_8) \\
B = {} & (-a_1a_{19}B_3a_9B_6 + a_1a_{13}B_3a_{15}B_6 + a_1B_3a_{15}a_{10}B_5 + a_1B_3a_{10}a_4B_{13} + a_1B_{11}a_{15}B_3a_4 \\
& - a_1B_3a_{16}B_5a_9 - a_1B_3a_{16}B_{10}a_4 - a_1a_9a_4B_3B_{14} + B_7B_3a_{19}a_9 - a_{13}B_3B_7a_{15} + a_{10}B_3B_7a_3a_{15} \\
& - a_{16}B_3B_7a_3a_9 - a_8B_8B_3a_{19} - a_8B_3B_{13}B_5 + a_8B_1B_3B_5a_{15} + a_8B_1B_{13}B_3a_4 - a_8B_4B_2B_5a_{15} \\
& - a_8B_4B_2B_{13}a_4 + a_8B_6a_2B_3a_{19} - a_8B_6B_3B_{13}a_3 - a_8B_6B_3a^2a_3a_{15} + a_8a_{16}B_3B_5a_2 + a_8a_{16}B_3B_8a_3 \\
& - a_{15}B_3B_9B_5 - a_4B_3B_9B_{13} + a_4B_1B_3B_9a_{15} - a_4B_2B_4B_9a_{15} - a_3B_3B_6B_9a_{15} + a_2a_4B_3B_9a_{16} \\
& + a_{13}a_{14}B_3B_8 + a_{14}B_3B_{10}B_5 - a_{14}B_1B_3B_5a_9 - a_{14}B_1B_3B_{10}a_4 + a_{14}B_2B_4B_5a_9 + a_{14}B_4B_2B_{10}a_4 \\
& - a_{13}a_{14}a_2B_3B_6 + a_{14}a_3B_6B_3B_{10} + a_{14}B_6B_3a^2a_3a_9 - a_{10}a_{14}a_2B_3B_5 - a_{14}a_{10}a_3B_3B_8 - a_{14}a_2a_4B_3B_{11} \\
& + a_9B_{12}B_3B_5 + a_4B_{12}B_3B_{10} - a_9a_4B_1B_3B_{12} + a_9a_4B_2B_4B_{12} + a_9a_3B_3B_6B_{12} - a_{10}a_4a_2B_3B_{12}) \\
& / (a_{14}B_5B_3a_9 - a_4B_3a_{15}a_8) \\
C = {} & (-a_1B_6B_3B_{10}a_{19} + a_1a_{13}B_6B_3B_{13} - a_1a_{19}a_9a^2B_3 + a_1a_{13}a_{15}a^2B_3B_6 + a_1a_{10}a_{19}B_3B_8 + a_1a_{10}B_5B_3B_{13} \\
& + a_1a_{15}B_{11}B_3B_5 + a_1a_4B_{11}B_3B_{13} - a_1a_{16}a_{13}B_3B_8 - a_1a_{16}B_{10}B_3B_5 - a_1a_9B_{14}B_3B_5 - a_1a_4B_{10}B_3B_{14} \\
& + a_{19}B_{10}B_3B_7 - a_{13}B_{13}B_3B_7 - a_{19}a_9B_7B_3B_1 + a_{13}a_{15}B_1B_3B_7 + a_{19}a_9B_7B_4B_2 - a_{13}a_{15}B_7B_4B_2 \\
& - a_2a_{10}a_{19}B_7B_3 + a_3a_{10}B_7B_3B_{13} + a_3a_{15}B_7B_3B_{11} + a_2a_{16}a_{13}B_7B_3 - a_3a_{16}B_7B_3B_{10} - a_3a_9B_7B_{14}B_3 \\
& + a_{19}a_8B_1B_8B_3 + a_8B_1B_5B_3B_{13} - a_{19}a_8B_2B_8B_4 - a_8B_4B_5B_2B_{13} + a_2a_8B_3B_5B_{14} + a_3a_8B_3B_8B_{14} \\
& + a_8a_2a_{19}a^2B_3B_6 - a_8a_3a^2B_{13}B_3B_6 - a_{19}B_9B_8B_3 - B_9B_5B_3B_{13} + a_{15}B_1B_5B_3B_9 + a_4B_1B_9B_3B_{13} \\
& - a_{15}B_2B_4B_5B_9 - a_4B_4B_9B_2B_{13} + a_2B_6B_9B_3B_{19} - a_3B_6B_9B_3B_{13} - a_3a_{15}B_6B_9B_3a^2 + a_{16}a_2B_5B_9B_3 \\
& + a_{16}a_3B_9B_8B_3 + a_4a_2B_9B_{14}B_3 - a_{14}a_{13}B_1B_8B_3 - a_{14}B_1B_8B_{10}B_5 + a_{14}B_4B_8B_2a_{13} + a_{14}B_4B_2B_{10}B_5 \\
& - a_3B_3B_8B_{11}a_{14} - a_2B_3B_5B_{11}a_{14} - a_{14}a_{13}a_2a^2B_3B_6 + a_{14}a_3a^2B_3B_6B_{10} + B_{12}B_8B_3a_{13} + B_{10}B_5B_3B_{12} \\
& - a_9B_1B_3B_{12}B_5 - a_4B_1B_3B_{12}B_{10} + a_9B_{12}B_4B_2B_5 + a_4B_2B_4B_{12}B_{10} - a_2a_{13}B_3B_6B_{12} + a_3B_3B_6B_{12}B_{10} \\
& + B_{12}a_9a_3a^2B_3B_6 - a_{10}a_2B_3B_5B_{12} - a_{10}a_3B_{12}B_8B_3 - a_4a_2B_{11}B_{12}B_3) / (a_{14}B_5B_3a_9 - a_4B_3a_{15}a_8).
\end{aligned}$$

$$E = (-a_1a_{19}a^2B_6B_3B_{10} + a_1a_{13}a^2B_6B_3B_{13} + a_1a_{19}B_8B_3B_{11} + a_1B_{13}B_5B_3B_{11} - a_1a_{13}B_8B_3B_{14}$$
$$- a_1B_5B_{10}B_3B_{14} - a_{19}B_1B_7B_3B_{10} + a_{13}B_1B_{13}B_3B_7 + a_{19}B_2B_4B_{10}B_7 - a_{13}B_{13}B_2B_4B_7 - a_2a_{19}B_7B_3B_{11}$$
$$+ a_3B_{13}B_{11}B_3B_7 + a_2a_{13}B_7B_3B_{14} - a_3B_3B_{10}B_{14}B_7 + a_{19}B_3B_1B_8B_9 + B_9B_1B_3B_{13}B_5 - a_{19}B_9B_4B_8B_2$$
$$- B_9B_4B_2B_{13}B_5 + B_9B_{14}B_3a_2B_5 + B_9B_{14}B_8B_3a_3 + a_{19}a_2a^2B_6B_3B_9 - B_9a_3a^2B_6B_3B_{13} - B_{12}B_1B_8B_3a_{13}$$
$$- B_{12}B_1B_3B_{10}B_5 + B_{12}B_4B_8B_2a_{13} + B_{12}B_4B_{10}B_2B_5 - B_{12}B_{11}B_3B_5a_2 - B_{12}B_{11}B_8B_3a_3 - B_{12}B_3B_6a_2a_{13}a^2$$
$$+ a_3a^2B_6B_3B_{10}B_{12}) / (a_{14}B_5B_3a_9 - a_4B_3a_{15}a_8)$$

The solution of Eq. (33) has the form

$$\Phi^* = \sum_{n=1}^{4} M_n e^{-k_n z} \tag{34}$$

$$\Psi^* = \sum_{n=1}^{4} H_{1n} M_n e^{-k_n z} \tag{35}$$

$$T^* = \sum_{n=1}^{4} H_{2n} M_n e^{-k_n z} \tag{36}$$

$$\phi_1^* = \sum_{n=1}^{4} H_{3n} M_n e^{-k_n z} \tag{37}$$

$$\psi_1^* = \sum_{n=1}^{4} H_{4n} M_n e^{-k_n z} \tag{38}$$

To get the displacement substituting Eqs. (37), (38) in (20) we get

$$u = \sum_{i=1}^{4} i\,a\,H_{3n} M_n e^{-k_n z} e^{i(\omega t + ax)} + \sum_{i=1}^{4} k_n\, H_{4n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{39}$$

$$w = \sum_{i=1}^{4} -k_n H_{3n} M_n e^{-k_n z} e^{i(\omega t + ax)} + \sum_{i=1}^{4} iaH_{4n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{40}$$

To get the stresses displacement substituting from Eqs (39) and (40) in (24)-(26) we get

$$\sigma_{xx} = \sum_{i=1}^{4} H_{5n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{41}$$

$$\sigma_{zz} = \sum_{i=1}^{4} H_{6n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{42}$$

$$\sigma_{xz} = \sum_{i=1}^{4} H_{7n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{43}$$

Dimensionless variables for the components of σ_i , τ_i

$$\sigma_3 = \eta_1 \Phi_{,z} + \eta_2 \Psi_{,z} \tag{44}$$

$$\tau_3 = \eta_3 \Phi_{,z} + \eta_4 \Psi_{,z} \tag{45}$$

Where, $\eta_1 = \frac{\alpha_1}{k_1\omega_1^2}$, $\eta_2 = \eta_3 = \frac{b_1\alpha_1}{\alpha k_1\omega_1^2}$, $\eta_3 = \frac{\gamma\alpha_1}{\alpha k_1\omega_1^2}$.

To get the solution of σ_3 and τ_3 substituting from Eqs. (34), (35) in (44) and (45)

$$\sigma_3 = \sum_{n=1}^{4} H_{8n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{46}$$

$$\tau_3 = \sum_{n=1}^{4} H_{9n} M_n e^{-k_n z} e^{i(\omega t + ax)} \tag{47}$$

Where,

$$H_{1n} = \frac{\begin{array}{l}\{-[a_{13}(a_{16}k_n^2 - B_{14}) - a_{19}(a_{10}k_n^2 - B_{11})][a_1 B_6 (k_n^2 - a^2) - B_7 (k_n^2 + f)] \\ -[(a_4 k_n^2 - B_5)(k_n^2 + f) - a_3 B_6 (k_n^2 - a^2)] \\ [(a_8 k_n^2 - B_9)(a_{16}k_n^2 - B_{14}) - (a_{14}k_n^2 - B_{12})(a_{10}k_n^2 - B_{11})]\}\end{array}}{\begin{array}{l}\{[a_2 B_6 (k_n^2 - a^2) - B_8 (k_n^2 + f)][a_{13}(a_{16}k_n^2 - B_{14}) - a_{19}(a_{10}k_n^2 - B_{11})] \\ -[(a_4 k_n^2 - B_5)(k_n^2 + f) - a_3 B_6 (k_n^2 - a^2)] \\ [(a_9 k_n^2 - B_{10})(a_{16}k_n^2 - B_{14}) - (a_{10}k_n^2 - B_{11})(a_{15}k_n^2 - B_{13})]\}\end{array}},$$

$$H_{2n} = \frac{-[a_1 B_6 (k_n^2 - a^2) - B_7 (k_n^2 + f)] + [a_2 B_6 (k_n^2 - a^2) - B_8 (k_n^2 + f)] H_{1n}}{[(a_4 k_n^2 - B_5)(k_n^2 + f) - a_3 B_6 (k_n^2 - a^2)]},$$

$$H_{3n} = \frac{(a_3 H_{1n} - a_1 - a_2 H_{1n})}{(k_n^2 + f)},\ H_{4n} = (\frac{-B_4}{B_3} H_{3n}),$$

$$H_{5n} = (\frac{\lambda}{\beta T_0}(k_n^2 - a^2) - \frac{2\mu a^2}{\beta T_0}) H_{3n} - H_{2n} + (\frac{b\alpha_1}{k_1 \omega_1^2 \beta T_0}) + (\frac{d\alpha_1}{k_1 \omega_1^2 \beta T_0}) H_{1n},$$

$$H_{6n} = [\frac{\lambda}{\beta T_0}(k_n^2 - a^2) + \frac{2\mu}{\beta T_0} k_n^2] H_{3n} - H_{2n} + (\frac{b\alpha_1}{k_1 \omega_1^2 \beta T_0}) + (\frac{d\alpha_1}{k_1 \omega_1^2 \beta T_0}) H_{1n},$$

$$H_{7n} = \frac{\mu}{\beta T_0}[-2 i a k_n H_{3n} - (a^2 + k_n^2) H_{4n}],\ H_{8n} = (-\eta_1 k_n - \eta_2 k_n H_{1n}),\ H_{9n} = (-\eta_3 k_n - \eta_4 k_n H_{1n}).$$

Boundary Conditions

We apply four boundary conditions for present problem at the plane surface z=0.

$$\sigma_{xx} = P_1 e^{i(\omega t + ax)} \tag{48}$$

$$\tau_3 = 0 \tag{49}$$

$$\sigma_3 = 0 \tag{50}$$

$$T = P_2 e^{i(\omega t + ax)} \tag{51}$$

Applying Eqs. (48)-(51) in (36), (41), (46) and (47) we get

$$\sum_{n=1}^{4} H_{5n} M_n = P_1 \tag{52}$$

$$\sum_{n=1}^{4} H_{9n} M_n = 0 \tag{53}$$

$$\sum_{n=1}^{4} H_{8n} M_n = 0 \tag{54}$$

$$\sum_{n=1}^{4} H_{2n} M_n = P_2 \tag{55}$$

To get $M_1, M_2, ..., M_4$, we can put Eqs. (52)-(55) in the matrix

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{pmatrix} = \begin{pmatrix} H_{51} & H_{52} & H_{53} & H_{54} \\ H_{91} & H_{92} & H_{93} & H_{94} \\ H_{81} & H_{82} & H_{83} & H_{84} \\ H_{21} & H_{22} & H_{23} & H_{24} \end{pmatrix} \begin{pmatrix} P_1 \\ 0 \\ 0 \\ P_2 \end{pmatrix} \tag{56}$$

Numerical Results

To study the effect of double porosity with the Rotation, we now present some numerical results. For this purpose, copper is taken as the thermoelastic material for which we take the following values of the different physical constants as [21].

$$\lambda = 7.7\times10^{10}\,N.M^{-2},\ \mu = 3.86\times10^{10}\,N.m^{-2},\ K = 3.86\times10^{3}\,N.s^{-1}.K^{-1},\ a = 2.5,\ \omega = -1,\ \alpha_t = 1.78\times10^{-5}\,K^{-1},$$
$$\rho = 8954Kg.m^{-3},\ C^{*} = 383.1\,J.Kg^{-1}K^{-1},\ T_0 = 293K,\ \tau_0 = 0.7,\ x = 0.5,\ \xi = -1,\ p_1 = 1\times10^{-5},\ p_2 = 2\times10^{-5},$$
$$t = 0.5,\ \Omega = 0.2,\ g = 9.8.$$

Following Khalili [17], the double porous parameters are taken as,

$$\alpha = 1.3\times10^{-5}\,N,\ b_1 = 0.12\times10^{-5}\,N,\ \gamma = 1.1\times10^{-5}\,N.m^{-2},\ \gamma_1 = 0.16\times10^{5}\,N.m^{-2},\ \gamma_2 = 0.219\times10^{5}\,N.m^{-2},$$
$$d = 0.1\times10^{10}\,N.m^{-2},\ b = 0.9\times10^{10}\,N.m^{-2},\ K_2 = 0.1546\times10^{-12}\,N.m^{-2},\ K_1 = 0.1456\times10^{-12}\,N.m^{-2}.$$

The numerical technique, outlined above, was used for the distribution of the real part of the temperature T the displacement components u, w the stress components σ_{xx}, σ_{xz} and the components of double porosity σ and τ for the problem. All the variables are taken in non-dimensional form the result.

Figures 1,2 show the comparison of the displacement u in the presence and absence of double porosity at ($\Omega = 0.2,\ \Omega = 0$). We find in Figure 1 that the displacement u increases at $\Omega = 0$ then decreases until it decay to zero, while u at $\Omega = 0.2,\ \Omega = 0$ decreases, then increases until it decay to zero. However, in Figure 3 the displacement decreases at ($\Omega = 0.2,\ \Omega = 0$) and takes the form of the wave until it decay to zero. Figures 3,4 illustrate the comparison of the displacement w in the presence and absence of double porosity at ($\Omega = 0.2,\ \Omega = 0$). We find that in Figure 3 the displacement w increases at ($\Omega = 0.2,\ \Omega = 0$) then increases until it decay to zero, but in Figure 4 the displacement w increases to a maximum value at z=0.5, and then decreases to a minimum value z=1.5 until it decay to zero at ($\Omega = 0.2,\ \Omega = 0$). Figures 5,6 explain the comparison of the temperature T in the presence and absence of double porosity at ($\Omega = 0.2,\ \Omega = 0$). We find in Figures 5,6 that the temperature T decreases in both two figures and satisfies the boundary condition at ($\Omega = 0.2,\ \Omega = 0$). Figures 7,8 demonstrate the comparison of the stress component σ_{xx} in the presence and absence of double porosity at ($\Omega = 0.2,\ \Omega = 0$). We find in Figure 7 that the stress σ_{xx} increases at $\Omega = 0.2$ more than $\Omega = 0$ to a maximum value at z=0.4, then decrease at the two cases and try to return to zero. In Figure 8 the stress σ_{xx} decreases at ($\Omega = 0.2,\ \Omega = 0$) then increases at the two cases and takes the form of wave and try to return to zero. Figures 9,10 demonstrate the comparison of the stress component σ_{xz} in the presence and absence of double porosity at ($\Omega = 0.2,\ \Omega = 0$). We find in Figure 9 that the stress σ_{xz} increases at $\Omega = 0$ more than $\Omega = 0.2$ than decreases until it decay to zero. Figure 10 illustrates that the stress σ_{xz} decreases to a maximum value at z=0.5, then increases to a minimum value at z=1.5 in the absence of double porosity, and takes the form of wave and try to return to zero. Figures 11,12 explain the comparison of the equilibrated stresses σ and τ in the presence of double porosity at ($\Omega = 0.2,\ \Omega = 0$). We find in Figures 11,12 that the equilibrated stresses σ and τ increase to a maximum value at z=0.2, and ($\Omega = 0.2,\ \Omega = 0$), then begin to decrease and take the form of wave and try to return to zero.

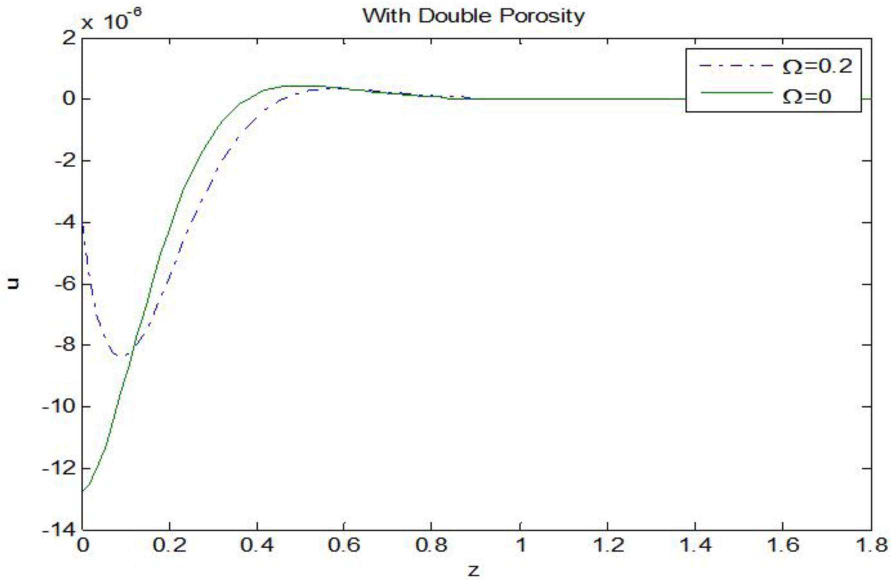

Figure 1: Distribution of the displacement u

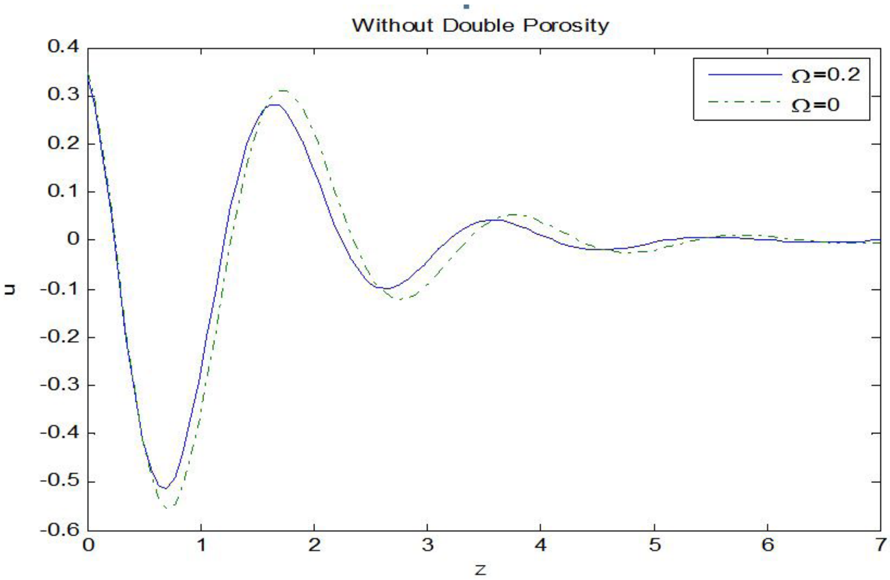

Figure 2: Distribution of the displacement u

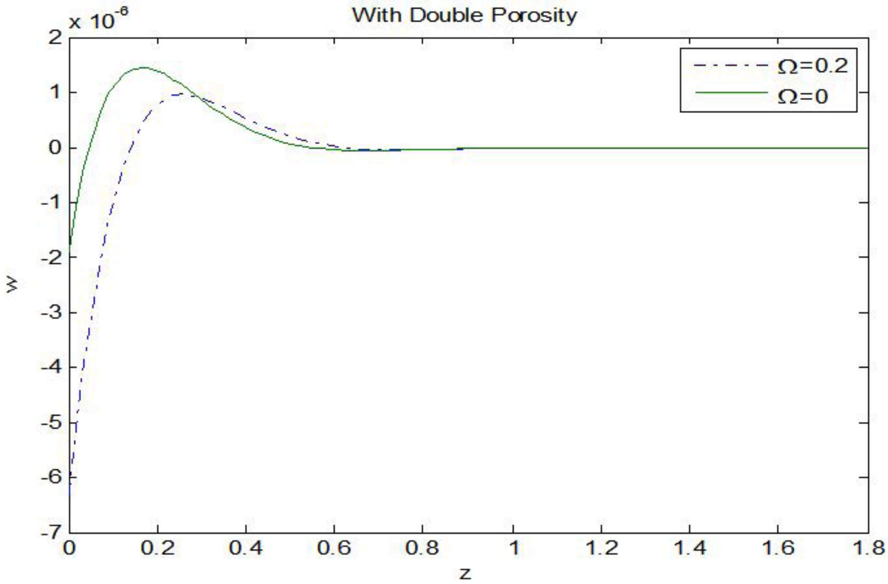

Figure 3: Distribution of the displacement w

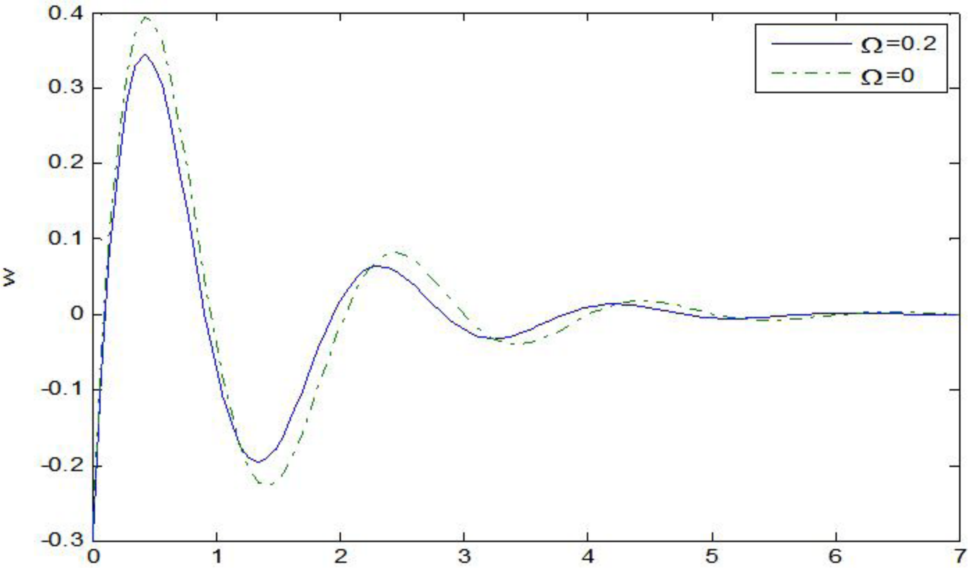

Figure 4: Distribution of the displacement w

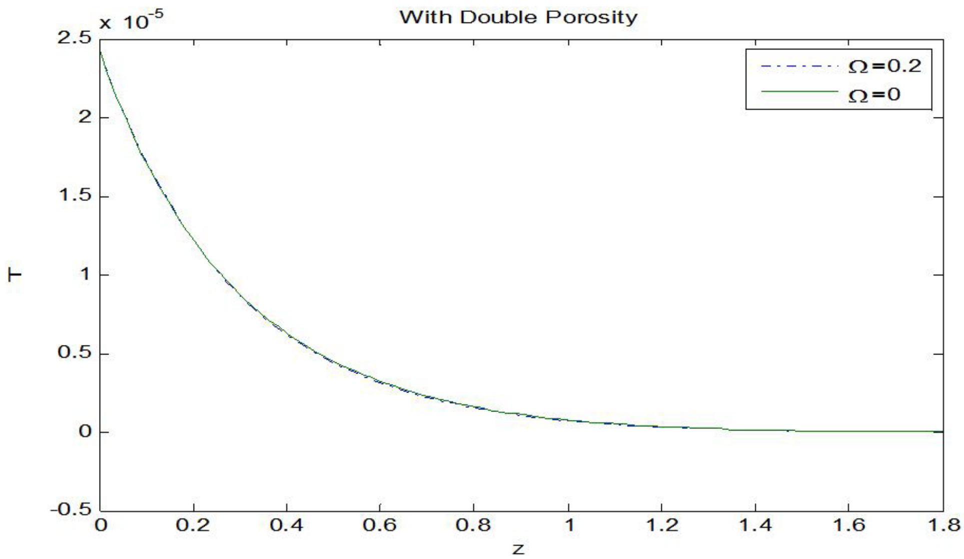

Figure 5: Distribution of the temperature T

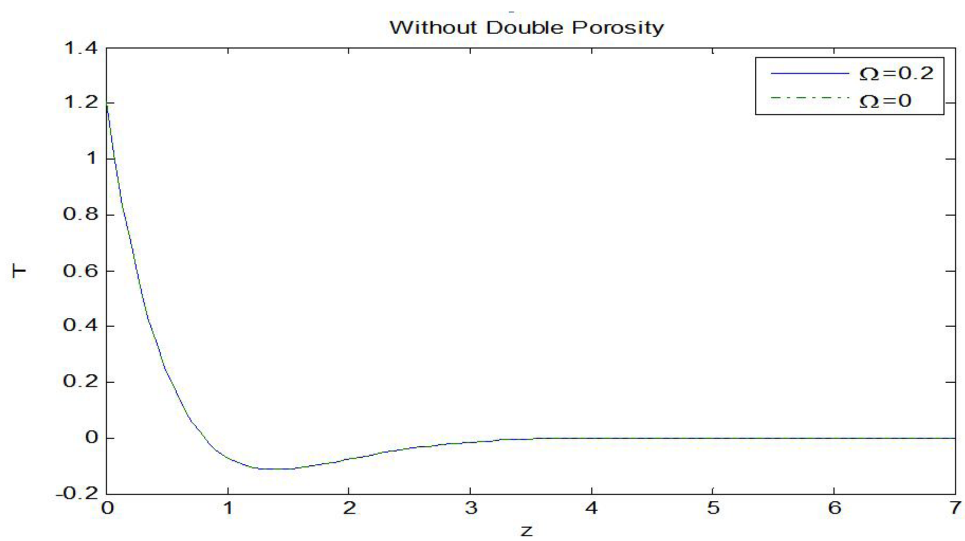

Figure 6: Distribution of the temperature T

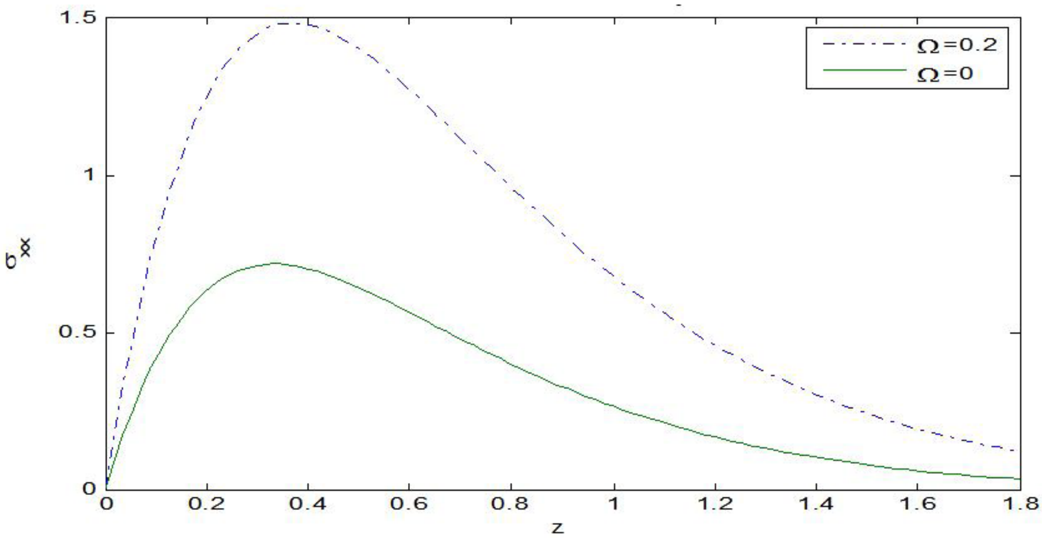

Figure 7: Distribution of the stress component σ_{xx}

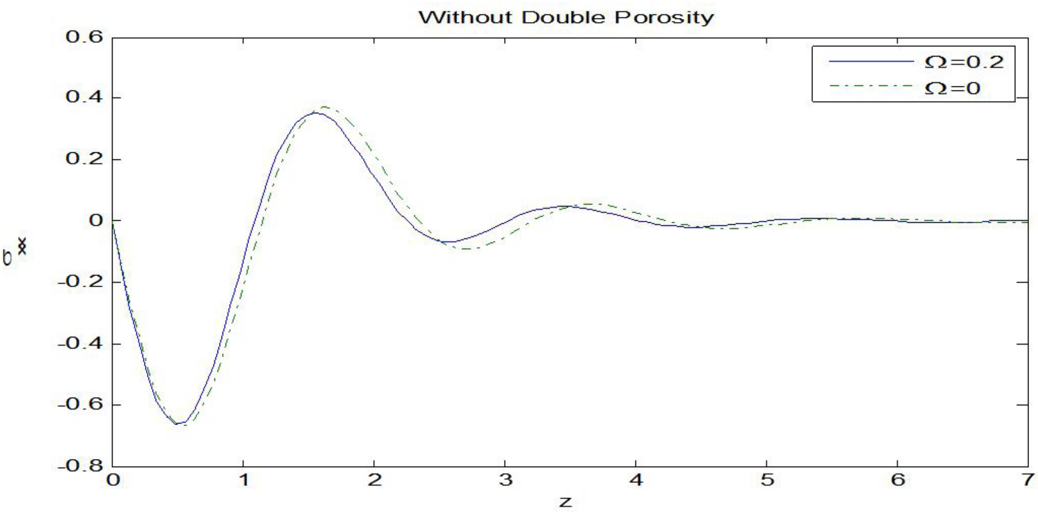

Figure 8: Distribution of the stress component σ_{xx}

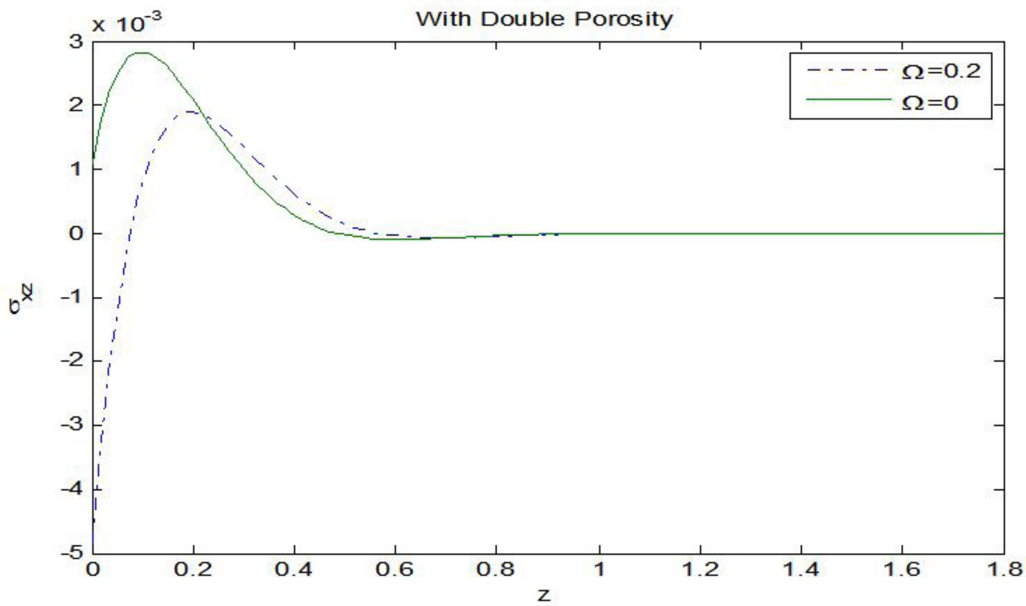

Figure 9: Distribution of the stress component σ_{xz}

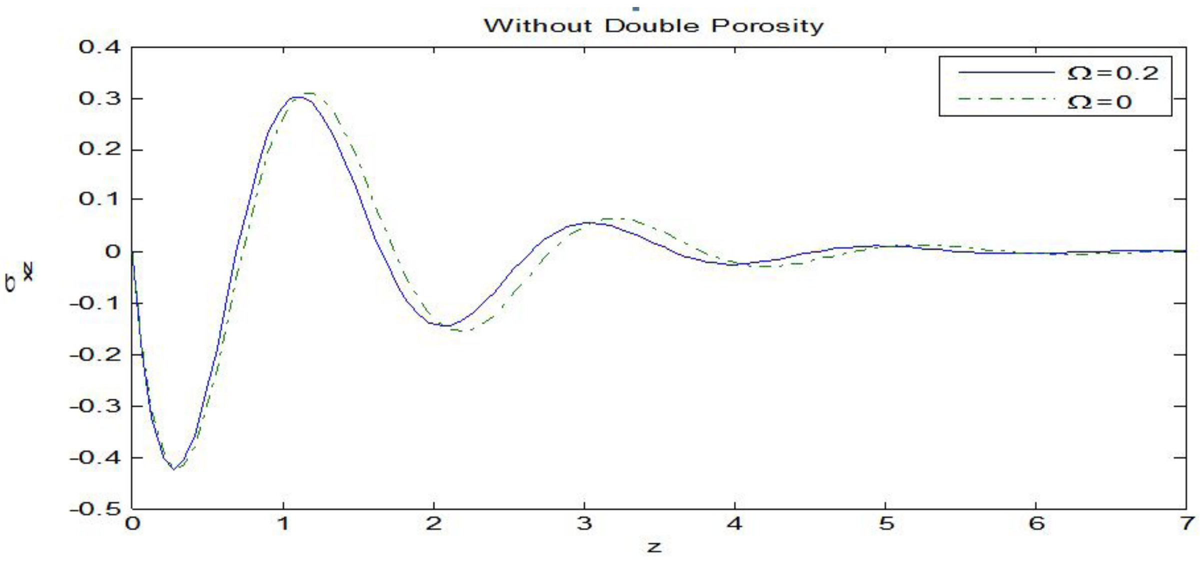

Figure 10: Distribution of the stress component $_{xz}$

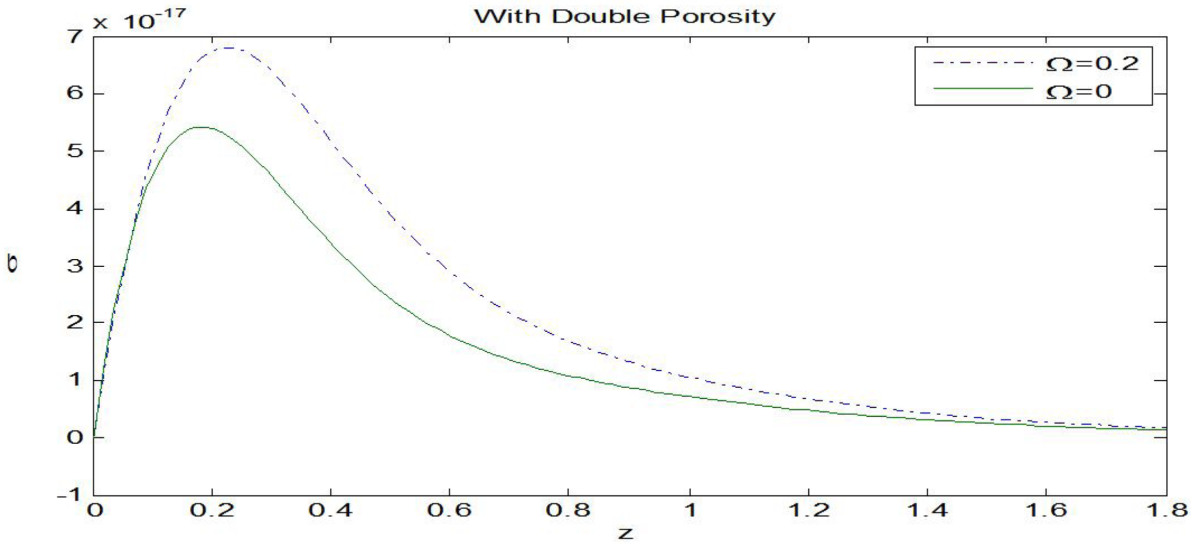

Figure 11: Distribution of the equilibrated stress σ

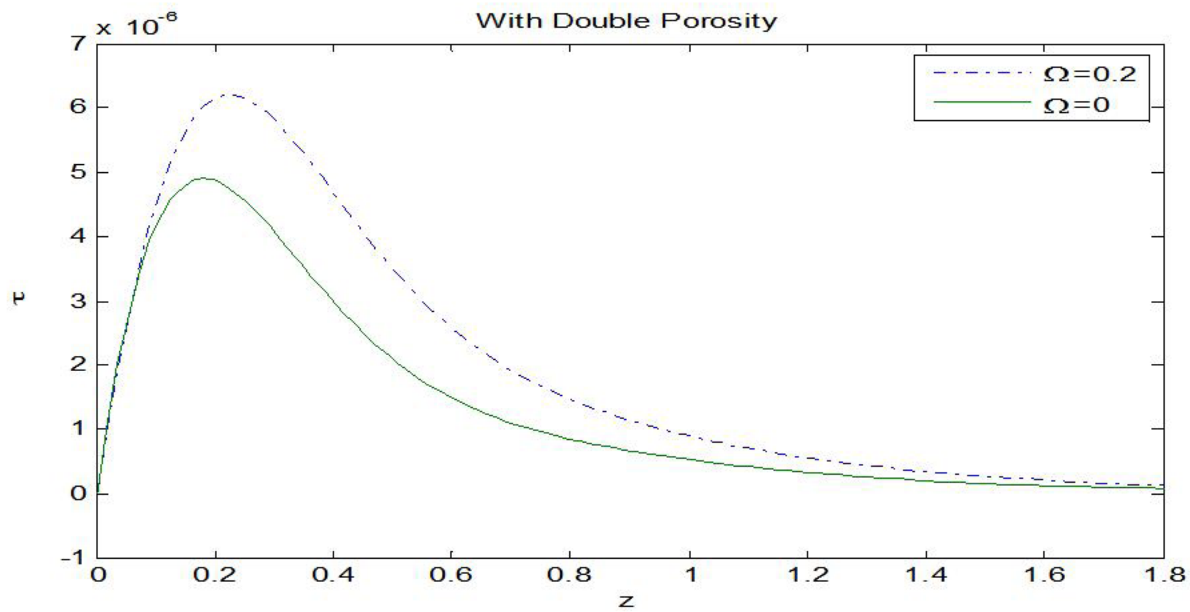

Figure 12: Distribution of the equilibrated stress τ

Conclusion

The figures obtained by comparing double porosity in the presence and absence of rotation, important phenomena are observed:

1. Analytic solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed.
2. The method that is used in the present article is applicable to a wide range of problems in hydrodynamics and thermoelasticity.
3. There are significant differences in the presence and absence of double porosity under the effect of rotation.
4. All the physical quantities satisfy the boundary conditions.
5. The value of all the physical quantities converges to zero, and all the functions are continuous.

The problem though theoretical, but it can provide useful information for experimental researchers working in the field of geophysics, earthquake engineering, along with seismologist working in the field of mining tremors and drilling into the crust of the earth.

Nomenclature

λ, μ Lame' parameters
μ, w Displacement vector
δ_{ij} Kronecker delta
ρ Mass density
c_e Specific heat at constant strain
σ_{ij} The stress tensor
v_1 The volume fraction field corresponding to pores and is the volume fraction field corresponding to fissures
Ψ, Φ The volume fraction fields corresponding to v_1 and v_2 respectively
K^* The volume coefficient of thermal expansion
$K \geq 0$ Thermal conductivity
k_1 and k_2 are coefficients of equilibrated inertia
T_0 Reference Temperature
τ_0 Relaxation time
$b, d, b_1, \gamma, \gamma_1, \gamma_2$ Constitutive coefficients
σ_i The equilibrated stress corresponding to v_1
τ_i The equilibrated stress corresponding to v_2
T The temperature change measured form the absolute temperature T_0
ω_{ij} Skew symmetric tensor called the rotation tensor
g Gravitational field

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