

# A Set of Axioms on Forces in the Mechanics of Rigid Bodies

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## Abstract

The following examinations focus on defining force, its characteristics, and the axiomatic definition of its effect. It is known that the Newtonian axioms define a few characteristics of force and the relationship among force and the parameters (coordinates) of acceleration in the case of movement (acceleration) of point-like bodies. It is also known that the static axioms of rigid bodies specify a few characteristics of the equilibrium of force systems. Recommendations are made in the study for establishing a complete system of axioms on forces. It is revealed that the recommended axiom system includes the Newtonian axioms in the case of system of point-like bodies, and axioms of statics in the case of the equilibrium of rigid bodies.

**Keywords:** Newtonian Axioms on the Forces; Axioms of Statics on the Forces; A Set of Axioms on Forces

## Introduction

Newton defined his system of axioms in „Principia” to describe the motion of (point-like) bodies (NEWTON, pp. 403-417) [1]. Newton describes the mechanical state of the body in the axiom system if no forces are acting upon it, and also defines how the magnitude and direction of force related with quantitative measures typical of the motion of the body when forces are acting upon it, and finally summarises how the magnitude and direction of forces relate when one body acts on a second body. Newton did not refer to the relation of two or more simultaneously occurring forces and how the joint effects of these could be determined. At the same time, he provided clear instructions regarding these two questions: forces are independent from each other when they occur and their effects add up according to the parallelogram rule. Newton, as his work was concentrated on the motion of (point-like) bodies, did not deal with the issue of balance.

Newton's axioms did not explicitly define that forces are vector quantities; this statement is discussed in *Corollary 1.* and 2. (NEWTON, p. 417 [4]).

*Notes.* In the literature usually the Newtonian definition of force caused the change of motion are used in the case of rigid body in equilibrium. But in equilibrium there is no any motion. The problems see for instance in JAMMER, pp. 221-222 [2]. In the case of equilibrium, inside the definition of force given the facts that force cause either motion or change of the size of a body (CHOLNOKY, pp. 33 [4]). But in the case of equilibrium of the rigid body either motion or change of size does not exist.

The axioms of statics are commonly defined for the equilibrium state of rigid bodies (SZILY, pp. 8-11, p. 23, CHOLNOKY, pp. 47-51 [3,4]). These axioms define the conditions of equilibrium for two or three forces, as well as the method of conversion of one equilibrium force system into another equilibrium force system. Finally, the (Newtonian) axiom of action-reaction is also classified as an axiom of statics, although the axiom system of statics does not deal with the occurrence of forces.

The axioms of statics do not explicitly define forces as vector quantities; this statement ensues from the axiom regarding the equilibrium of three forces.

Generally, neither of the axiom systems deals with the occurrence of forces, although both include the axiom, which clearly defines the number of forces and their relationship to each other which can be interpreted as the model of effects arising between two bodies. The two axiom systems apply to two different mechanical systems: the first one is intended to describe the motion of (point-like) bodies while the second one is intended to describe the equilibrium of rigid bodies.

The fact that any triplet of numbers forms a vector is not unambiguous. There is reference that the axioms record the attributes (commutativity, associativity) of vector addition when interpreting the vector spaces and attributes of simultaneous operations with vectors and real numbers (HALMOS [5]). The fact that force is a vector quantity and the joint effect of more forces is equivalent to the effect of the vectorial sum of more forces does not ensue from the fact that force is used to describe the interaction of two bodies. The study examines the two axiom systems, then an axiom system is compiled which includes the attributes of forces arising in mechanics, the operations regarding the mechanical state of bodies, i.e. the „method of use” of force.

*Notes.* Usually the axiom of parallelogram rule is taken into consideration when the Newtonian system of axiom of forces is presented, see APPEL pp. 100-103 (but the axioms are formulated not for force, but for kinematics), and TEODORESCU pp. 45-49 [6,7]. In the problem of static there are another approach: instead of axioms the “elementary operations” are introduced, see APPEL pp. 150-151, and TEODORESCU pp. 95-99 [6,7]. In the literature to analyse the equilibrium of rigid body usually the axiom of liberation from constrains is applied, see TEODORESCU pp. 167 [7]. The first result in the topic of the axioms of the forces in the mechanics of rigid bodies was presented on the conference “IMT Oradea” 2017. marc. 25-27, LÁMER [8].

The Newtonian system of axiom deals with characterisation of forces in the case of the motion of (point-like) bodies, the axioms of statics deal with characterisation of forces in the case the equilibrium of rigid bodies. Only one of the axioms is the same: the axiom of action and reactions. The definitions of force are different. The standpoint of our disquisition is the definitions of the force in the case of equilibrium and motion must be the same, the mathematical characterization of the forces must be unified, but the method of calculation of effects of forces must be characterised the individual physical phenomena.

## The Newtonian Axioms

Newton's axioms apply to motion of bodies, more specifically to the state of motion of (point-like) bodies (NEWTON) [1].

Before presenting the axioms, a definition of force, of body, and of mechanical state is required for which the axioms apply to.

Newton defines force as the effect acting upon the body. “*Definition 4.* Impressed force is the action exerted on a body to change its state either of resting or of moving uniformly straight forward.” (NEWTON, p. 405 [1].)

Newton uses the term matter instead of body. “*Definition 1.:* Quantity of matter is a measure of matter that arises from its density and volume.” (NEWTON, p. 403.) (Presently referred to as the product [1].)

Newton defines the „initial” mechanical state as rest or uniform motion. See i.e. *Definition 4.*

The Newtonian axioms are the following.

**The First Newtonian Axiom (Law 1):** “Every body perseveres in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by forces impressed.” (NEWTON, p. 416 [1].)

This axiom applies to the state when no forces are acting upon the body; in other words, no other body is acting upon it.

**The Second Newtonian Axiom (Law 2):** “A changes in motion is proportional to the motive force impressed and takes place along the straight line in which that force impressed.” (NEWTON, p. 416 [1].)

This axiom, for a force impressed on a body, is currently defined as follows: the acceleration (**a**) of an object arises in the same direction as the force (**F**) causing it while its magnitude is proportional to the magnitude of the force and inversely proportional to the mass (*m*) of that object.

This axiom describes the relationship between the data determining force (magnitude, direction and sign) and data describing the motion change of the body (magnitude, direction and sign). This also means that this axiom defines the data describing the force at the same time: magnitude, direction and sign. However, this axiom does not describe how force arises. This is because gravitational forces act upon objects in a Newtonian mechanical system and the value and direction of these forces is determined by the relative position, mass and universal laws of gravity. On the one hand, the second axiom describes that if force acts upon an object then it changes the motion of the object, on the other hand it also describes how the change of the motion can be calculated. The second axiom connects only the force and its consequence and it does not deal with the simultaneous effect of more forces. This has to be examined separately.

**The Third Newtonian Axiom (Law 3):** “To any action there is always an opposite and equal reactions; in other words, the actions of two bodies upon each other are always equal and opposite in direction.” (NEWTON, p. 417 [1].)

This axiom defines that if an effect arises while two bodies act upon each other, then these are essentially two „effects”, or two forces. At the same time, it also defines how the data describing two forces – magnitude, direction and sign – relate to each other. It does not provide explicit information of the line of action, but it covertly assumes identical line of action in the subsequent explanation.

This axiom characterises how the force arises. Implicitly, it „follows” the first axiom. The second axiom characterises the numeric value of force and its effect, i.e. effect of one of the two occurring forces on the examined object. Therefore, the third axiom „precedes” the second axiom.

**The Fourth and Fifth Newtonian Axioms:** In fact, it defines not as an axiom but rather as *Corollary 1.* and *2.*

*Corollary 1.* “A body acted on by [two] forces acting jointly describes the diagonal of a parallelogram in the same time in which it would describe the sides if the forces were acting separately.” (NEWTON, p. 417 [1].)

In order to prove *Corollary 1.* Newton does not apply the previous axioms, but a logical reasoning that the forces are independent from each other; therefore, it is irrelevant whether on which side of the parallelogram the body moves along first and second towards the endpoint of the diagonal. This reasoning uses the independence of forces, the attributes of which were not individually postulated by Newton. We also refer to the fact that the parallelogram rule is regarded as an axiom in linear vector spaces (see e.g. (HALMOS [5]).

So, *Corollary 1.* which is generally regarded as The Fourth Newtonian Axiom (see e.g. BUDÓ, p. 42 [9]) includes two independent statements. On the one hand, it states that the effect of forces acting upon a body are independent, on the other hand, they sum up as vectors. Since the Newtonian axioms apply to (point-like) bodies, forces act upon them in one point; there are no conceptual obstacles for the vectors of forces to sum up.

Similarly to the summation of forces, Newton states in an individual *Corollary 2.* that a force can be separated to two oblique forces.

*Corollary 2.* “And hence the composition of a direct force *AD* out of any oblique forces *AB* and *BD* is evident, and conversely the resolution of any direct force *AD* into any oblique forces *AB* and *BD*. And this kind of composition and resolution is indeed abundantly confirmed from mechanics.” (NEWTON, p. 418 [1].)

In order to prove this, Newton used the condition torque of forces on one point and the balance of parallel forces placed on two opposite ends of the balance beam of the scale.

The following statements can be made in connection with Newtonian axioms:

- The axioms do not include information regarding the occurrence of force, i.e. the axioms do not determine the generation of force.
- The second axiom, more specifically its definition in the form of  $F = ma$ , enables the measurement of mass and weight force (see e.g. BUDÓ, pp. 38-42 [9]).
- The axioms apply to the motion of bodies that can be described with shifting. Instead of (point-like) bodies, they can also be regarded as extensive objects (for instance rigid or deformable solid bodies) but occurring forces only act in the centre of gravity and the bodies cannot rotate around their centre of gravity.
- The axioms do not deal with the movement of extensive, e.g. rigid or deformable solid, bodies. These have to be examined separately.
- The axioms do not refer to equilibrium and the effect of the equilibrium force systems on motion.
- The axioms do not apply to the rotation, deformation and fracture of bodies.
- The axioms do not include the occurrence of constraint forces, and neither the fact that in the case of a constraint created by a surface, the constraint force is orthogonal to the surface.

Newton's axioms primarily apply to the motion of bodies, the bodies are point-like, and the followings are excluded:

- deformation of bodies,
- fracture of bodies,
- equilibrium of bodies.

## Axioms of Statics

The axioms of statics apply to the equilibrium of rigid bodies (see e.g. SZILY , pp. 8-11, p. 23, and CHOLNOKY , pp. 47-51) [3,4]. The axioms of statics are explained based on CHOLNOKY [4].

The axioms primarily apply to forces. The fourth axiom includes a statement specifically concerning rigid bodies acting upon each other. Therefore, we provide the basic terms – body, rigid body, force, data of force, equilibrium – prior to the description of the axioms.

CHOLNOKY does not provide a definition of the body. He provides a definition that physics and thus mechanics deal with such phenomenon during which „matter and chemical composition of bodies do not change” (CHOLNOKY, p. 20. [4]). This provision does not apply to either the state of matter or the internal structure of the body, or more precisely to its „stability” of states of matter. Beside the stability of matter and chemical composition, the body can also be very different: gas, liquid, heaped body or solid. Transient forms also exist, such as thick liquid (paste), the plastic deformation of solid bodies, or plastic fluid (see e.g. LÁMER [10,11]). The definition of the body does not only depend on matter and chemical stability but also on internal structure.

**Definition of rigid body:** „Rigid body is such an abstract body, which does not change its shape and measurements under any external pressure.” CHOLNOKY, p. 20. [4].

**Definition of Force:** „Force is such an abstract term, which can unambiguously describe and provide the effect of a body changing the motion or size of another body”. (CHOLNOKY, p. 33. [4].)

The unambiguous characterisation is a „self-evident” requirement in an axiomatic structure therefore we do not consider it necessary to particularly specify this. Furthermore, mechanics today does not only include changes in motion and strain but also intactness of the body. Damage to the body, as a material, is examined within the framework of mechanics e.g.: the formation of cracks, fractures, plastic deformation, furthermore the damage of the body as a (load bearing) structure, and finally the loss of (positional and structural) stability in the mechanical state of the body. In fact, these phenomena do not reveal force but rather the effect of force on the body. Therefore, these can be omitted from the explanation of the force.

**Data of Force:** „Force is characterised by four data: point of application, line of action, sign and magnitude.” (See: CHOLNOKY, p. 36. [4].)

In order to interpret the state of equilibrium, one term is required: the state of rest of body.

**Definition of State of Rest:** „another object, observed from a coordinate system fixed to the chosen object, we find that practically none of the coordinates of its points change in time. Then we say about the other object that it is in a *state of rest*.” (See: CHOLNOKY, p. 22; highlighted there [4].) (The question of reference is dealt with later.)

**Definition of Equilibrium Force System:** „A force system is in equilibrium when they act on an object originally in the state of rest and the object remains in a state of rest.” (see: CHOLNOKY, p. 43. [4]). A force system in equilibrium is „the equilibrium force systems” (see: CHOLNOKY, *ibid.* [4]).

At the same time, rest is a special case of state of motion. The interpretation of equilibrium does not change if instead of rest, we consider the general – rigid body-like – state of motion of the body. Accordingly, we redefine the term of equilibrium force system.

**The (abstract) Definition of Equilibrium Force System:** A force system is in equilibrium if the force system acts upon a rigid body, which is in a determined (state) of motion, and the body remains in the same (state) of motion.

**Remark:** Persistence of strain is not a sufficient condition for interpreting equilibrium force systems since the equilibrium force system acting upon a body in the state of rest can change the shape of the body.

The axioms of statics are the following.

**The First Axiom of Statics:** “Two forces of identical magnitude and line of action, with opposite sign are in equilibrium” (CHOLNOKY, p. 47. [4]).

This axiom captures, by providing two forces in equilibrium, that equilibrium force systems exist. `

This axiom defines the equilibrium of two forces irrespective of point of application: the point of application of forces is simply not included among the conditions of equilibrium. Consequently, two forces do not have to act upon in the same point; the condition of equilibrium only depends on concordance of line of action (beyond the identical magnitude and opposite sign).

**The Second Axiom of Statics:** “Three forces with one point of intersection of their action's lines are in equilibrium if and only if the vectors of three forces constitute a closed, continuous vector polygon (triangle).” (CHOLNOKY, p. 48. [4].)

The axiom implicitly applies the fact that the force vectors are present as independent entities, as well as the fact that they can be summed as vectors. These are conditions for identifying the sum of three force vectors with the force equal that of the simultaneous effect of three forces.

The closed, continuous vector polygon indicates that the three forces must be in one plane.

**The Third Axiom of Statics:** “If we connect an equilibrium force system to another equilibrium force system or an equilibrium subsystem is taken away from an equilibrium system, the obtained force system remains in equilibrium.” (CHOLNOKY, p. 49. [4].)

This axiom states how another equilibrium force system can be obtained from one equilibrium force system. Only a strictly equilibrium force system can be used to create an equilibrium force system: either two equilibrium force systems are „added”, or a partial equilibrium force system constituting a part of the equilibrium force system is „subtracted”. Equilibrium force system cannot be created from non-equilibrium force system. This can be formulated as equilibrium and non-equilibrium force systems are independent from each other.

**The Fourth Axiom of Statics:** “If forces arise in pairs when two rigid bodies act upon each other, the pair of forces have the same line of action, the same magnitude and opposite sign.” (CHOLNOKY, p. 50. [4].)

This axiom defines only those forces arise in pairs. It does not include information on either the number of force pairs or the point of application occurring with force pairs and neither does it refer to whether the two rigid bodies are motionless or are in motion in the moment of occurrence of force. It does not specify whether motionless bodies begin to move when force pairs arise or do the arising forces change motion in the case of moving bodies or not.

The two forces, in The Fourth Axiom of Statics, between two rigid bodies occurring as a consequence of an effect, are in equilibrium according to The First Axiom of Statics.

The following statements can be made regarding the axioms of statics:

- The axioms do not include information on the occurrence of forces, i.e. the axioms do not determine the occurrence of forces.
- The axioms are formally applied to the equilibrium of rigid bodies. In fact, the first three axioms of statics include conditions suitable for determining equilibrium irrespective of the fact whether forces act upon rigid or deformable bodies or the forces are simply given in space. The fourth axiom of statics includes reference to rigid bodies, as long as the aforementioned axiom makes constraints regarding the force pairs occurring during the interaction of two rigid bodies.
- The axioms do not refer to motion and the effect of motion on the equilibrium force system.
- The axioms do not deal with the effect of forces occurring between bodies on the state of equilibrium and motion.
- The axioms do not apply to the motion (shifting), rotation, deformation and fracture of bodies.
- The axioms do not include the development of constraint forces or the fact that the constraint force is orthogonal to the surface in the case of constraint created by a surface.

The axioms of statics are primarily applied to rigid bodies at rest, i.e. exclude:

- deformation of bodies,
- motion of bodies,
- fracture of bodies.

We examine such tasks within statics, where the system of motionless, rigid bodies in equilibrium rearrange due to the effect of a given force system and reach a motionless state of equilibrium, but the path to this is imagined as the rearrangement of rigid body systems occurs “without” motion until equilibrium is reached. Bodies do not gain either velocity or acceleration during the rearrangement. Equilibrium is reached infinitely slowly.

## Relationship of Newtonian Axioms and Statics Axioms

The two axiom systems apply to the mathematical description of the behaviour of two different mechanical systems. For this reason, the two axiom systems differ from each other significantly. At the same time, there are identical and similar elements also.

**THE FIRST NEWTONIAN AXIOM:** captures the mechanical state when there are no acting forces. The axioms of statics do not explicitly deal with this state. The equilibrium state of a force system can exist in the state of rest, in the state of moving uniformly straight line, as well as in the case of an accelerating body. This is not captured in the axioms of statics.

**THE SECOND NEWTONIAN AXIOM:** captures that the force interpreted as a model of an effect of a body and its actual effect on the motion (the change from the state of rest or from constant (in velocity) movement in straight line to the state of acceleration) are in one direction. According to our current definition, there is linear proportionality between the force triggering the change in the state of motion and the acceleration describing/characterising the change of the motion and the factor of proportionality is the mass. There is no relationship between force and its effect in statics. Primarily because in statics especially in the statics of rigid bodies force does not have any (tangible) effect on rigid bodies. What an axiom of statics could capture, is that the direction, magnitude and sign of the force arising between two rigid bodies can and must be determined exactly from the existing equilibrium (if the interpreted mechanical system is suitable for this; presuming the definition of statically determinate system).

**THE THIRD NEWTONIAN AXIOM:** captures the principle of action and reaction just like **THE FOURTH AXIOM OF STATICS**.

The *Corollary 1.*, regarded as **THE FOURTH NEWTONIAN AXIOM** states that the effect of forces acting upon a point-like body is independent. The *Corollary 2.*, regarded as **THE FIFTH NEWTONIAN AXIOM**, states that forces acting upon a point-like body sum up as vectors. The axioms of statics do not make these statements explicitly at the same time, but **THE THIRD AXIOM OF STATICS** use these statements.

The Newtonian Axioms, primarily because of the 4<sup>th</sup> and 5<sup>th</sup> Axioms, include that a rigid body should do two independent motions simultaneously, furthermore that the two motions can be examined separately as well as together. The independence of forces and a vectorial summation (if forces act on one point) in statics does not provide guidance on mechanical state – equilibrium. At the same time, the 3<sup>rd</sup> axiom of statics captures how equilibrium force systems relate to each other: they are independent, can be examined separately and together, the equilibrium exists separately and together as well.

Both axiom systems capture knowledge regarding forces for a special mechanical system.

## Criteria of Establishing the Axiom System

The behaviour of bodies under forces is determined by knowledge based on experience, and from them abstracted fundamental theorems that is axioms.

The mechanical terms used in the definition of axioms, such as force, body, inertia, and motion have to be interpreted. One part of the terms, primarily ones related to kinematics are not interpreted assuming that they are known.

We follow the under mentioned principles when setting up axiom system:

The axioms must record

- how the force occurs,
- what data describe (characterise) the force,
- how the occurring force(s) can be described,
- what the effect of two forces on each other like,
- what mathematical object force is,
- what state exists if there is force and if there is no force,
- how the forces characterising a mechanical system (equilibrium force system) can be determined in the case of an „elementary”, or a „typical” system,
- how can be created or derived from one specific (let understand: equilibrium) force system an another specific (that is equilibrium) system, different from beginning one.

The axioms are classified into four large groups.

- The first group summarises the knowledge regarding the occurrence of forces.
- The second group includes interpretations of forces as mathematical objects.
- The third group records the effect of forces on bodies.
- The fourth group describes the relationship between forces and their effects.

## A System of Axioms Related to Forces

We wish to interpret force as the effect arising between bodies therefore we provide definitions of the different types of body.

**The First Definition (Interpretation of body):** *The body* is a collection of several such particles, which keep own physical and chemical identity, they move jointly, but they relative arrangement of any two particles of the body (the relative position of the particles, namely their sequence) can be changed. In general, body fill a closed domain of the Euclidean space, we usually attributed the domain with different mechanical characteristics, primarily mass density, but the mechanical effect does not can be interpreted mathematically as a continuous map of the Euclidean space which assigns one the closed domain to another closed domain.

In this sense the gaseous, the liquid, the heap (of grains) and the massive, firm forms of the material are bodies.

**The Second Definition (Interpretation of solid body):** A *body* is considered to be *solid* if the relative arrangement of any two particles of the body (the relative position of the particles that is their sequence) remains the same under mechanical effect. In general, body is considered to be a closed domain of the Euclidean space, and the mechanical effect can be interpreted as such a continuous map of the Euclidean space which assigns the closed domain to another closed domain. The position of solid body is given by the position of every points of the domain.

In this sense the massive, firm form of the material is solid body.

**The Third Definition (Interpretation of deformable solid body):** A *solid body* is considered to be *deformable*, if the distance between any two points of the body can change under mechanical effects.

Physically every solid body is deformable.

**The Fourth Definition (Interpretation of rigid body):** The rigid body is such a model of the solid body in which model the distance between any two points of the body is constant under mechanical effects. The position of rigid body is given by the position of its reference point *and* situation of the body in the space. Mechanically the rigid body is characterized by mass density *and* inertia tensor. The reference point is usually the centre of gravity.

**The Fifth Definition (Interpretation of material point):** Material point is such a model of the solid body, in which the position of the body is given by its reference point, and the mass of the body is „concentrated into” this point. The reference point is usually the centre of gravity. We generally use the term point-like body as well. In this model there is no information about the position of points of the body.

## Axioms Related to the Occurrence of Force

The first we interpret force.

**The Sixth definition (Interpretation of force):** The *force* is the mechanical model of two interacting solid bodies.

**Remark:** According to *The Sixth definition*, force is not a tangible entity but rather an abstract term. We will still phrase as if force was some kind of tangible thing and say that force was created, force effects, force acts or force is applied on a body.

The first axiom records how force occurs.

**The First Axiom (The axiom of interpretation of force):** An effect occurring between two solid bodies is modelled as two forces: *one* body acts upon another body and vice versa, the *other* body acts upon the first one.

So, when two bodies act upon each other, in the case of one kind of effect, a pair of forces occur. In the case of various effects, forces occur in pairs when two bodies act upon each other; as many pairs as many effects occur between the two bodies.

With two given bodies,  $A$  and  $B$ , and an existing effect between them (e.g. they are in contact in a point, or gravitational attraction exists between them) the effect of body  $B$  on body  $A$  is denoted by  $F_{AB}$ , the effect of body  $A$  on body  $B$  is denoted as  $F_{BA}$ . Since The First Axiom only records that there is an effect, so it is denoted by a „neutral” notation, by letter  $\mathcal{F}$ . If there are more effects between body  $A$  and  $B$  (e.g. there are contact in several points, or gravitational attraction and electrical attraction and repulsion also exists between them), then the forces occurring in pair differ from each other in accordance with more effects; the forces  $\mathcal{F}_{AB}^1$  and  $\mathcal{F}_{BA}^1$ , as well as  $\mathcal{F}_{AB}^2$  and  $\mathcal{F}_{BA}^2$  occurring in contact point 1 *and* 2, and the forces  $\mathcal{F}_{AB}^G$  and  $\mathcal{F}_{BA}^G$ , as well as  $\mathcal{F}_{AB}^E$  and  $\mathcal{F}_{BA}^E$  occurring as an effect of gravitational attraction *and* electrical attraction and repulsion.

The second axiom records the data characterising the force:

**The Second Axiom (The axiom of data of force):** Force is characterised by the point of application, line of action, sign, and magnitude.

We characterise the occurring forces in the third axiom.

**The Third Axiom (The axiom of action and reaction):** The two forces occurring between two bodies as a result of effects between two bodies are characterised as follows: the line of action of two forces is in coincidence, the magnitude of two forces is the same, and their sign is opposite. We say that there is an „action” and a „reaction” occurring for each effect between two bodies.

The Third Axiom does not have „provision” on the point of application of forces. The point of application is interpreted at different points in various modelling. For example, in the case of gravitational bodies, gravitating force is connected to the centre of gravity of the gravitating body. Forces arising during the gravitational effect are far from each other in celestial body revolving around a central celestial body. Contact forces are in the same contact points in the case of contacting bodies.

It must be recorded how a force arising in the absence and presence of another force or several forces.

The fourth axiom records the method of effects of two or more forces, more specifically the independence of two or more forces from each other.

**The Fourth Axiom (The axiom of independence of forces):** The individual forces act upon independently. Let given three bodies,  $A$ ,  $B$  and  $C$ . The effect of body  $B$  on body  $A$  when there is no body  $C$ , denoting by  $F_{AB}(\Phi)$ , and  $F_{AB}(C)$ , if the body  $C$  is present, when body  $A$  and  $B$  act upon each other. The independence of forces means that  $F_{AB}(\Phi)$  is identical with  $F_{AB}(C)$ , i.e. the effect of body  $A$  and  $B$  on each other is not influenced by body  $C$ . Naturally, the axiom also applies to cases when more  $C_i$  ( $i = 1, 2, \dots, n$ ) bodies are also present beyond body  $A$  and  $B$ . In this case, the force  $F_{AB}(\Phi)$  arising without the  $C_i$  ( $i = 1, 2, \dots, n$ ) bodies is identical with the  $F_{AB}(C_i)$  ( $i = 1, 2, \dots, n$ ) force arising in the presence of any  $C_i$  ( $i = 1, 2, \dots, n$ ) body, with the  $F_{AB}(C_i, C_j)$  ( $i, j = 1, 2, \dots, n, i \neq j$ ) force arising along with any two,  $C_i$  and  $C_j$  ( $i, j = 1, 2, \dots, n, i \neq j$ ) bodies and with the  $F_{AB}(C_i, C_j, \dots, C_l)$  ( $i \neq j, i \neq l, \dots, j \neq l, i, j, \dots, l = 1, 2, \dots, n$ ) forces arising along with any combination of bodies  $C_i, C_j, \dots, C_l$  ( $i \neq j, i \neq l, \dots, j \neq l, i, j, \dots, l = 1, 2, \dots, n$ ).

## Force as a Mathematical Object

The fifth axiom records what mathematical object is force.

**The Fifth Axiom (The axiom of choosing mathematical object):** Force is a vector quantity.

Since the axiom captures that effect is modelled as vector, effect – the force – is marked with bold letter  $\mathcal{F}$  hereinafter. In the example above the effect of body  $B$  on body  $A$  is marked  $\mathcal{F}_{AB}$ , the effect of body  $A$  on body  $B$  is marked  $\mathcal{F}_{BA}$ , the magnitude of forces is marked with  $\mathcal{F}_{AB}$  and  $\mathcal{F}_{BA}$ , respectively.

With the occurrence of two or more effects, i.e. two or more force occurrence the fact must be recorded that if two or more forces are simultaneously interpreted, how those jointly act upon the body. This is recorded in the sixth axiom.

**The Sixth Axiom (The axiom of joint effect of forces):** Forces with *identical point of application* acting upon independently sum up as vectors.

THE SIXTH AXIOM states that two forces with *identical point of application* sum up as vectors: their joint effects – their geometrical attributes – can be determined by vector addition. The point of application of the „summed” force is the common point of application of forces, line of action and magnitude is determined based on the rules of vector addition.

**Remarks:** 1. The „summation” (more precisely the addition of the effects) of two forces with non-intersected *and* non-parallel line of action is not done by vector addition, but by the addition of screws (see e.g.: BALL [12]). 2. Force summation makes sense if the changes occurring as a result of force can also be „summed”. This requirement is met in kinematics because on the one hand acceleration happens as an effect of force, on the other hand the two accelerations of the same mass point can be vectorially added. This requirement is also met in the case of equilibrium of the rigid body, because the force does not occur any effect on rigid body.

## The Effect of Force on Bodies: Motion and Immobility

The following possibilities have to be separated in this section.

- No force acts upon the body
- Constraint can also exist beyond force: it forces one or several points of the body to remain in a specified geometrical place
- One force acts upon the body and no constraint exists
- More forces act upon the body, no constraint exists and the body is in a state of rest: the force system is in equilibrium (it can be regarded as the definition of the equilibrium force system)
- The existence of equilibrium force system must be assured by an axiom
- If there are two or more given equilibrium force systems, how another equilibrium force system can be created from them
- Some forces act upon the body and some constraints exist

As the first step of the programme above, we provide the fact in the seventh axiom that a body is at rest without force. This requires a few terms.

We start out from the fact that *force is a model of two bodies acting up on each other*, so the absence of a body is required for a body not have this effect. So, a body without force is such a model construction in which the examined body is separated from the Universe in order to make the body independent from the Universe and all the attention is focused on the body. Hereinafter, two cases are possible, and their combination as the third: another body acts or other bodies act on the body separated from the Universe in the same way, or the Universe itself acts on the body, and both. In the first case we completely separate the examined bodies and the Universe, in the second and third cases the Universe is only present as the environment of the examined body with bodies acting on it. The effect of the Universe from the aspect of interpreting forces can be attributable to the effect of one body (or possibly more bodies). Therefore, the determination (interpretation) of force as model does not require regarding the Universe and instead carry out our examinations in such an „environment” in which a condition can occur where no acting forces are present.

The state of rest or motion can only be interpreted relative to the other bodies. In the three-dimensional space formally we need three reference bodies; practically we need only the marked points of the reference bodies (as an environment). We usually will mention reference point, but we will mean a (marked) point of the reference body.

**The Seventh Definition (Interpretation of state of rest):** The body is in *the state of rest* if the distances of all points of the body remain unchanged from the three given reference points of the environment in time.

Motion can be and is to be interpreted simultaneously with the state of rest.

**The Eighth definition (Interpretation of state of motion):** The body is in *the state of motion*, if the distance of at least one of the points of the body changes from the three given reference points of the environment in time.

**Remarks:** 1. Time is not interpreted. The revolution of the Earth around the Sun is commonly used to interpret time (see e.g.: CHOLNOKY, p. 18-20 [4]). 2. Elementary geometric relationships can reveal that instead of three points of the environment, the three coordinates of the (all) points of the body can be used to determine the motionless and the motion of the body in the environment in time 3. During the interpretation of the state of rest and motion it is implicitly presumed that given reference points are in the state of rest but those can also be in motion themselves relative to other points considered in the state of rest. This issue is dealt with independently.

Points providing the reference can be in the state of rest or can perform movement relative to another reference points. Therefore new reference points have to be chosen. In order to avoid confusion, primary, secondary, tertiary etc. reference points will be mentioned. It has to be decided in the case of primary reference points whether the secondary reference point used for determining the state of rest of primary reference points are in the state of rest or in motion relative to a new – tertiary – group of reference points. Selection of the reference points can continue infinitely in order to determine the state of rest and motion. This can be terminated if the body in the *absolute* state of rest are defined. This latter does not exist as everything in the Universe moves. What remains is a system of bodies that can be regarded as practically in the state of rest. This will simply be named environment in the state of rest. This is what Newton called inertial system, system in the absolute state of rest.

Examination of the state of rest and motion reveals (GALILEI, Kepler and Descartes, see BUDÓ, p. 37), that the state of rest, the state of motion with constant velocity in a straight line and the absence of the force are connected [9,13]:

- a body with no force acting upon it can be in the state of rest and can also perform motion with constant velocity in a straight line relative to other bodies in the state of rest, furthermore
- a body with no force acting upon it can also be in the state of rest and can also perform motion with constant velocity in a straight line relative to other bodies in the state of motion in straight line with constant velocity.



So, both the state of rest and of motion with constant velocity in a straight line of the body *and* the state of rest and of motion with constant velocity in a straight line of the environment providing the reference are interchangeable *as long as no force acts upon the body*. In the seventh axiom not only the state of a body has to be revealed if no force acts upon it but also that such environments (coordinate systems) exist for describing the condition where no force acts upon the body.

**The Seventh Axiom (The axiom of environment in the state of rest and the state of rest of body):** There exists such a system of environments (coordinate systems) that its elements being in the state of rest or performing motion with constant velocity in a straight line relative to each other in which (i.e. in environment) the body, with no forces acting upon it is in the state of rest or performs motion with constant velocity in a straight line.

Therefore, THE SEVENTH AXIOM states that

- a class of environments exists (simplifying the Universe in order to identify the place of the examined body) which has equivalent elements,
- the elements of this class of environment are in the state of rest relative to each other or perform motion with constant velocity in a straight line relative to each other,
- the body left alone (no forces acting upon it) in these environments is also in a state of rest itself or performs motion with constant velocity in a straight line relative to its environment.

Hereinafter, the specified environment is called *inertial system*. Environment is given as a *coordinate system* for practical calculations.

In Terrestrial conditions, a coordinate system linked to Earth can be regarded as inertial system in the first approach. Application of coordinate systems linked to Earth leads to calculations with adequate accuracy in the case of static calculations. There is such forms of motion, for which the evolution of Earth has to be taken into consideration (for example determining the trajectory of a bullet, explanation of planar pendulum rotating around a vertical axis, interpreting the direction of liquid vortex in the case of outflow at the bottom of a container). In the case of more complex forms of motion the coordinate system linked to the so-called fixed stars can be regarded as inertial system. (See more at BUDÓ, p. 37. [9].)

Hereinafter, we presume that the coordinate system used for describing the phenomenon is inertial one. For the sake of simplified phrasing we do not normally specify either in the case of body or inertial system that the body or inertial system can be in the state of rest and can perform motion with constant velocity in a straight line but only that it is in a state of rest. So, if we state that a body is in the state of rest, then what we mean by this is that there is such an inertial system in the state of rest in which the body is in the state of rest.

THE SEVENTH AXIOM applies to the condition when no force acts upon the body; in other words, no other body acts upon it. A separate axiom is used for capturing if a force acts upon a body. At the same time, not only force can act on a body but a constraint preventing the movement of its points for which force – constraint force – is assigned.

**The Ninth Definition (Interpretation of elementary constraint (support)):** A body preventing the displacement in one direction of a point of the body analysed is called elementary constraint (support).

**The Tenth Definition (Interpretation of constraint force (support force)):** A force acting upon a body under the effect of constraint is called *constraint force (support force)*. The contact force acts in the line of prevented by constraint displacement at the point fixed by the constraint.

**The Eleventh Definition (Interpretation of equilibrium force system):** Assume there is a body in the state of rest, which is not under a constraint. Let's act multiple forces upon the body, under which the body remains at rest! In this case, the forces acting upon the body are called *equilibrium force systems*.

The effect of force acting upon the body has to be examined separately for cases without and with the presence of constraint. The case of force acting upon the body under no constraint is captured in the eighth axiom.

**The Eighth Axiom (The axiom of the motion):** A body in the state of rest, upon which one force is acted, and is not under the effect of any constraint (compared to the motionless environment) does not remain motionless, it gains (accelerated) motion.

This axiom only records the fact that bodies without constraint, and under effect of a force are to start to move. The question "what the movement is" – the movement depends on the type of body – can be recorded by some individual axioms. At the same time, it can be excluded that the motion is with constant velocity in a straight line since such an inertial system can also be chosen for describing the condition of body in the state rest, from which the motion seems to be with constant velocity a straight line. Because of this we wrote in parentheses that the motion is accelerated.

In the case of constraints, forces arising from the constraints also act on the body under forces. This fact is captured by the ninth axiom.

**The Ninth Axiom (The axiom of arising constraint force (support force)):** A body, with no forces acting upon it, and is under some constraints has no force arising from constraints. A body, with some forces acting upon it, and is under some constraints have forces arising from constraints. The line of action is given by constrain, the magnitude and the sign of constraint forces can be determined from the condition that the forces and constraint forces acting up on the body are in equilibrium.

THE NINTH AXIOM captures the conditions in which constraint forces arise or do not arise. Furthermore, it does not define the magnitude and the sign of the constraint forces, only give the condition to determine them. Since we did not define the general condition of equilibrium, THE NINTH AXIOM cannot include the method of determining constraint forces in general.

Using the term of equilibrium force system, the mechanical state of body under forces and constraints can be defined.

*The First consequence.* Let given a body, which is under the effect of more forces and more constraints and the forces as well the constraint forces jointly

- form an equilibrium force system, then the body is motionless,
- do not form an equilibrium force system, then the body begins to move.

**Remark:** If the condition of an equilibrium force system is known, then the first consequence can be used exactly for determining constraint forces.

Interpretation of the equilibrium force system does not ensure that an equilibrium force system exists and does not provide a method for determining the equilibrium force system. The tenth axiom provides the necessary and sufficient conditions for the equilibrium for the cases of two forces and also provides the existence an equilibrium force system at the same time.

**The Tenth Axiom (The axiom of equilibrium for two forces; the axiom of existence of equilibrium force systems):** Two forces with identical line of action, identical magnitude and opposite sign are in equilibrium.

THE TENTH AXIOM records that an equilibrium force system exists by providing two forces in equilibrium.

THE TENTH AXIOM is practically THE FIRST AXIOM OF STATICS (slightly rephrased). Therefore, the statements made after THE FIRST AXIOM OF STATICS also apply to this axiom. Namely, THE TENTH AXIOM determines the equilibrium of two forces irrespective of point of application: the points of application simply are not present among the conditions of equilibrium. Consequently, two forces do not have to act upon in the same point, the condition of equilibrium only depends on the concordance of the lines action (beyond the identical magnitude and opposite sign).

*The Second consequence.* The two forces arising during action-reaction between two bodies are in equilibrium.

*The Third consequence.* The two bodies however, do not have to be in equilibrium; see e.g. the gravitational forces among moving Planets and the Sun.

The fact that the two forces arising during action-reaction are in equilibrium can also be defined as the  $\mathbf{F}_{AB}$  and  $\mathbf{F}_{BA}$  forces arising during the interaction of  $A$  and  $B$  bodies are opposite each of other.

The opposite of force  $\mathbf{F}$  will be marked  $\mathbf{F}'$ . According to THE TENTH AXIOM, the force system consisting of forces  $\mathbf{F}$  and  $\mathbf{F}'$  is in equilibrium that is:  $\{\mathbf{F}, \mathbf{F}'\} \doteq \{\mathbf{0}\}$ . According to THE TENTH AXIOM forces  $\mathbf{F}$  and  $\mathbf{F}'$  are opposite each of other. Consequently, not only force  $\mathbf{F}'$  is the opposite of force  $\mathbf{F}$ , but force  $\mathbf{F}$  is also the opposite of force  $\mathbf{F}'$ , that is  $(\mathbf{F}')'$  is equivalent to force  $\mathbf{F}$ ;  $(\mathbf{F}')' \doteq \mathbf{F}$ .

THE THIRD AXIOM can be redefined with the term of the opposite of force.

THE THIRD' AXIOM: Force and its opposite arise as a result of effect between two bodies.

Since force is a vector quantity according to THE FIFTH AXIOM, if we identify force with a vector  $\mathbf{F}$ , then the opposite of the force is identified with the vector with the opposite sign  $-\mathbf{F}$ , that is:  $\mathbf{F}' = -\mathbf{F}$ . In this marking „-“ is not „subtraction“, but the sign of creating an opposite force (vector). It is known from vector algebra that the composition of vectors and one opposite vectors is equivalent to the difference of system of vectors and the opposite vector  $\{\mathbf{A}_p, \mathbf{F}'\} \doteq \{\mathbf{A}_p, -\mathbf{F}\}$ .

By using the term of opposite force, the term of opposite of force system is introduced. If there is a force system  $\{\mathbf{A}_i\}$ , the opposite of all forces gives the opposite force system:  $\{\mathbf{A}'_i\} \doteq \{\mathbf{A}_i\}'$ . According to THE TENTH AXIOM, the composition of a force system and its opposite force system gives an equilibrium force system:  $\{\{\mathbf{A}_i\}, \{\mathbf{A}'_i\}\} \doteq \{\mathbf{0}\}$ .

The eleventh axiom records the method creating an equilibrium force system from another equilibrium force system.

**The Eleventh Axiom (The principle (axiom) of superposition of equilibrium force system):** If an equilibrium force system is connected to an equilibrium force system or an equilibrium subsystem is taken away from an equilibrium force system, then the obtained force system is in equilibrium.

THE ELEVENTH AXIOM states, how two equilibrium force systems,  $\{\mathbf{A}_i\} \doteq \{\mathbf{0}\}$  and  $\{\mathbf{B}_j\} \doteq \{\mathbf{0}\}$ , can be used to create an equilibrium force system: the two independent equilibrium force systems are joined, i.e. the two joined force systems are regarded as one force system. The obtained force system is also in equilibrium:  $\{\{\mathbf{A}_i\}, \{\mathbf{B}_j\}\} \doteq \{\mathbf{0}\}$ . Furthermore, THE ELEVENTH AXIOM states that if there is an equilibrium force system,  $\{\mathbf{C}_i\} \doteq \{\mathbf{0}\}$ , which can be separate into two subsystems in such a way that one is in equilibrium,  $\{\mathbf{C}_i\} \doteq \{\{\mathbf{A}_i\}, \{\mathbf{B}_j\}\}$  and  $\{\mathbf{B}_j\} \doteq \{\mathbf{0}\}$ , then leaving the equilibrium subsystem out of the original equilibrium force system, the remaining subsystem remains in equilibrium:  $\{\mathbf{A}_i\} \doteq \{\mathbf{0}\}$ .

**Remark:** A statement regarding the equilibrium of three forces beyond the equilibrium of two forces is commonly expressed as an axiom in statics.

**The axiom of Equilibrium of Three Forces:** Three forces with one point of intersection of their action's lines are in equilibrium if and only if the vectors of the three forces forms a closed vector polygon.

THE AXIOM OF EQUILIBRIUM OF THREE FORCES is not independent of the above-mentioned axiom system; it follows from the *Axiom of forces acting upon each other (THE FIFTH AND SIXTH AXIOM)* and the *Principle of superposition of equilibrium of force systems (THE ELEVENTH AXIOM)*.

Motion and equilibrium have been separated in the axioms so far. The question arises on how these two phenomena relate to each other and whether one condition changes in presence of another. The twelfth axiom records that the state of equilibrium and motion are independent mechanical states.

**The Twelfth Axiom (The axiom of independence of equilibrium and motion):** If an equilibrium force system acts upon a body in motion in such a way the physical parameters of the body remain (practically) unchanged than the motion is not influenced by the equilibrium force system and vice versa, an equilibrium force system acting upon a body in the state of rest does not change (the equilibrium force system remains), if the body being in the state of rest begin to move in such a way the physical parameters of the body remain (practically) unchanged.

One condition appears twice in *THE TWELFTH AXIOM*: "In such a way the physical parameters of the body remain (practically) unchanged". In the case of a point-like body, the parameter in question is mass. This does not normally change with either motion or when the equilibrium force system is acting upon it. In the case of rigid body, the parameter in question is the mass and the inertia tensor. These parameters do not normally change with either motion or when the equilibrium force system is acting upon it. In the case of deformable solid bodies, the equilibrium force system does not change mass but rather the distribution of the mass and the inertia tensor of the body can also change. Consequently, motion can change and the equilibrium of the force system can also change – the equilibrium can come to an end – as a result. This explains the „condition“.

**Remark:** *THE TWELFTH AXIOM* records the independence of motion and equilibrium. At the same time, it does not separate what mechanical phenomenon occurs in equilibrium: contact of two bodies, deformation of a body, rearrangement the particle of a heaped body, friction between two bodies, etc. This will be defined by the laws related to determining force.

The axiom of motion and the state of rest declares when force arises between two bodies, the bodies can set in motion. This possibility is generally excluded during the examination of equilibrium. Accordingly, it must be captured that the force arising in the equilibrium system reaches its value extremely slowly and the bodies do not accelerate during this. This is phrased in the thirteenth axiom.

**The Thirteenth Axiom (The axiom of effect of equilibrium force):** The forces arising in equilibrium must be arisen infinitely slowly, and do not act velocity and acceleration.

**Remark:** If required, we take into consideration that the system of bodies rearranges, but motion (that is the acceleration) as a mechanical condition is disregarded. We can also state that the bodies come into being in equilibrium by an extremely slow rearrangement.

## The relationship existing between force and its effect

The following axioms describe the relationship among various forces and their effects:

The relationship between various forces and their effects are given by the following axioms. We relied on literature to mathematically define forces (BUDÓ [9,14], and TEODORESCU, pp. 39-43. [7]).

**The Fourteenth Axiom (The axiom of gravitational forces):** Force arising between two gravitating bodies (Budó, p. 30. [9]):

$$\mathbf{F}_{grav} = -\gamma \frac{m_1 m_2}{r^2} \frac{\mathbf{r}}{r}, \quad (1)$$

where:

- $\mathbf{F}_{grav}$  – gravitational force;
- $m_i$  – mass of the body, ( $i = 1,2$ );
- $\mathbf{r}$  – position vector connecting the two centres of gravity of bodies;
- $\gamma$  – universal gravitational constant.

**The Fifteenth Axiom (The axiom of weight force, that is the gravitational force on the surface of the Earth):** The weight force acting upon bodies with weight is (Budó, p. 40. [9]):

$$\mathbf{G}_w = m\mathbf{g}, \quad (2)$$

where:

- $G_w$  – weight force on the surface of the Earth;
- $m$  – mass of the body;
- $\mathbf{g}$  – gravitational acceleration on the surface of the Earth.

**The Sixteenth Axiom (The axiom of constraint force):** A force arises at a contact point in a rigid body under constraint is orthogonal to the rigid surface of constraint. Its magnitude can be determined from the conditions of equilibrium (Budó, p. 56. [9]).

**The Seventeenth Axiom (The axiom of frictional force):** Let press a force  $\mathbf{G}$  the point-like (rigid) body to a rigid surface constituting the constraint. Let act a force  $\mathbf{F}$  in the tangent plane to the surface in the contact point of the body and the surface. Than a frictional force acting upon the body opposite in direction of the force  $\mathbf{F}$  is (BUDÓ, p. 60. [9]):

$$\mathbf{F}_{fr} \leq -\mu G \frac{\mathbf{F}}{F}, \quad (3)$$

where:

- $\mathbf{F}_{fr}$  – frictional force;
- $\mathbf{F}$  – force acting upon the body in the tangent plane to the surface;
- $\mu$  – friction coefficient between the body and the surface providing the constraint;
- $G$  – magnitude of force orthogonally pressing the body against the surface constituting the constraint.

**The Eighteenth Axiom (The axiom of acceleration of the material point):** A material point with mass  $m$  (without constraint) under force  $\mathbf{F}_{acc}$  accelerates with acceleration  $\mathbf{a}$  in the same direction as the force  $\mathbf{F}_{acc}$ , the coefficient of proportionality is mass  $m$  (BUDÓ, p. 38. [9]):

$$\mathbf{F}_{acc} = m\mathbf{a}, \quad (4)$$

where:

- $\mathbf{F}_{acc}$  – force triggering the acceleration;
- $m$  – mass of the material point;
- $\mathbf{a}$  – acceleration of the body under force  $\mathbf{F}_{acc}$ .

The acceleration, angular acceleration of the rigid body does not have to be individually postulated, those can be derived from the acceleration of the system of material points.

**The Nineteenth Axiom (The axiom of force arising between electrically charged particles):** Force arising between two electrically charged point-like bodies is (the Coulomb's law, BUDÓ, p. 16. [14]):

$$\mathbf{F}_{electr} = K \frac{e_1 e_2}{r^2} \frac{\mathbf{r}}{r}, \quad (5)$$

where:

- $\mathbf{F}_{electr}$  – force arising between electrically charged bodies;
- $e_i$  – electric charge of body, ( $i = 1,2$ );
- $\mathbf{r}$  – position vector connecting the centres of gravity of the two bodies;
- $K$  – universal attraction constant (the constant of the Coulomb's law).

**The Twentieth Axiom (The axiom of force arising between magnetically charged particles):** Force arising between two electrically charged point-like bodies is (the Coulomb's law, BUDÓ, p. 16. [14]):

$$\mathbf{F}_{magn} = C \frac{p_1 p_2}{r^2} \frac{\mathbf{r}}{r}, \quad (6)$$

where:

- $\mathbf{F}_{magn}$  – force arising between magnetically charged bodies;  
 $p_i$  – magnetic (mono)charge of body, ( $i = 1,2$ );  
 $\mathbf{r}$  – position vector connecting the centres of gravity of the two bodies;  
 $C$  – universal magnetic attraction constant.

The axiom above can be considered as the axiom applying to magnetic monopoles but can also be considered as the force arising between magnetic dipoles.

**The Twenty-first Axiom (The axiom of forces acting upon electrically charged particles moving in an electromagnetic field):**

Forces acting on electrically charged point-like particle moving in the electromagnetic field is (Lorentz-force, BUDÓ, p. 144. [14]):

$$\mathbf{F}_{Lorentz} = Q \left( \mathbf{E} + \frac{1}{c} [\mathbf{v}\mathbf{B}] \right), \quad (7)$$

where:

- $\mathbf{F}_{Lorentz}$  – force acting on electrically charged particle moving in electromagnetic field;  
 $Q$  – electric charge of the body;  
 $\mathbf{v}$  – velocity vector of the body;  
 $\mathbf{E}$  – electric field;  
 $\mathbf{B}$  – magnetic field;  
 $c$  – speed of light.

**The Twelfth definition (Interpretation of force-displacement diagram and its elements):** Assume that a force acts on a deformable solid body fixed against a motion, the value of which increases from zero to the maximum value. This process is referred to as uploading. A decreasing force from its maximal value to zero is referred to as downloading. During uploading and downloading the solid body, normally the displacement of the point of application of the loading force, is measured. The force–displacement curve is referred to as uploading and downloading curve.

The following behavioural forms of the deformable solid body can have distinguished based on the relative position of the uploading and downloading curve.

**The Thirteenth definition (Interpretation of behaviour):** When the uploading and downloading curves overlap each other, the behaviour is *elastic*. If the uploading (and at the same time the downloading) curve is straight line during elastic behaviour, then we speak of linearly elastic behaviour. When the uploading and downloading curves differ from each other and the downloading curve (nearly) parallel with the ascending branch of the uploading curve, then the behaviour of the body is plastic. Both elastic and plastic behaviour is constant in time: displacement is unchanged with unchanged force. If the displacement changes (increases) with constant force, then we speak of viscous (time-dependent) behaviour.

**The Twenty-second Axiom (The axiom of elastic, plastic and viscous forces):** During the elastic, plastic and viscous behaviour of the deformable solid body, elastic, plastic and viscous internal forces arise in the body.

Connections related to elastic, plastic and viscous behaviour exist between stress and deformation. Description of these exceeds the objectives of this study. Instead of providing equations, we refer to related literature [15-17].

## The Relationships between the Newtonian Axioms and the Set of Axioms on Forces as well as the Axioms of Statics and The Set of Axioms on Forces

In the paper we gave the systematic definitions of different body (in the viewpoint of mechanical behaviour of the body), a unified definition of the force in the case of mechanical behaviour of body, and a set of axioms on forces.

Correspondence between the Newtonian axioms and the set of axioms on forces are the following:

- Newton's 1<sup>st</sup> axiom  $\Leftrightarrow$  7<sup>th</sup> and 8<sup>th</sup> axiom of forces,
- Newton's 2<sup>nd</sup> axiom  $\Leftrightarrow$  18<sup>th</sup> axiom of forces,
- Newton's 3<sup>rd</sup> axiom  $\Leftrightarrow$  3<sup>rd</sup> axiom of forces,
- Newton's 4<sup>th</sup> axiom  $\Leftrightarrow$  4<sup>th</sup> axiom of forces (independence of forces)
- Newton's 5<sup>th</sup> axiom  $\Leftrightarrow$  5<sup>th</sup> axiom of forces (forces are vector quantities), and 6<sup>th</sup> axiom of forces (joint effect of forces).

Correspondence between the axioms of statics and the set of axioms on forces are the following:

- The 1<sup>st</sup> axiom of statics  $\Leftrightarrow$  10<sup>th</sup> axiom forces,
- The 2<sup>nd</sup> axiom of statics  $\Leftrightarrow$  derive from the 4<sup>th</sup>, 5<sup>th</sup>, 6<sup>th</sup> and 10<sup>th</sup> axioms forces,
- The 3<sup>rd</sup> axiom of statics  $\Leftrightarrow$  11<sup>th</sup> axiom forces,
- The 4<sup>th</sup> axiom of statics  $\Leftrightarrow$  3<sup>rd</sup> axiom forces.

By the presentation of correspondence between the Newtonian axioms and the set of axioms on forces, as well as by the presentation of correspondence between the axioms of statics and the set of axioms on forces we show, that the set of axioms on forces includes both the Newtonian axioms and the axioms of statics.

The set of axioms on forces use a unified definition of force, and do not need to use the Newtonian definition of force caused the change of motion in the case of rigid body in equilibrium, and do not need to use the definition of force caused change of size in the case of rigid body in equilibrium. The set of axioms on forces unified the Newtonian's axioms and the axioms of statics.

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